Present Thursday - 11.5 #26, 37, 38

9:55: Katie M Alyssa N Nicole N Rachel R Hannah R Chelsea S
11:00: Emma R Jamie S Erin S Jamie S Victoria S Abbey S

Ch. 11 Review - Counting Techniques

Homework '[illegible]
#1-4, 6, 8, 15-18, 19, 20, 21, 24, 25-28

Present Monday: #17, 21, 27

9:55: Molly S Amy S Jared T Briana T Rachel W Jena W
11:00: Kelly S Jeffrey S Kayla T Kelly T Calli V Molly W

Name tags Please :)

Quick quiz today on 11.3 after presentations.

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8)
7 coins tossed - count ways to get
at least one of each (H & T)

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$2^7 = 128 \rightarrow$ at least one of each $= 128 - 2 = 126$

not "at least one of each"
= either no heads or no tails

H H H H H H H
T T T T T T T

After reviewing the correct solution (below), write your score on the back of your quiz.

0 = no progress at all; just rewrote problem
0.5 = false start, not based on relevant principles
1 = false start, but sustained effort with some relevant principles
1.5 = significant mistake(s), or significant misunderstanding(s)
2 = mistake near the end or could not finish; also excessive reliance on calculator or 'brute force' methods
2.5 = trivial mistake (e.g. arithmetic error), but work is mostly correct
3 = correct answer and work

$C(26,5) = 65,780$

To test for defective ~~parts~~ 6, samples of ~~three~~ 5 gas pedals are chosen from a shipment and ~~tested~~. If the shipment contains two defective gas pedals (out of 32 total), how many ways could a three-item sample not contain any of the defective gas pedals?

30 good, two defective.
To have no defective pedals, we'd have to choose all 3 from the 30 good ones. So $C(30,3) = \frac{\overset{5}{\cancel{30}} \cdot 29 \cdot 28}{\cancel{3 \cdot 2} \cdot 1} = 4{,}060$ ways.

Optional (not for credit; comes from Sec. 11.5): How many of the three-item samples would contain at least one defective gas pedal?
$C(32,3) - C(30,3) = 4960 - 4060 = 900$ ways.
$900/4960 \approx 18\%$ of the samples will find a defective pedal.

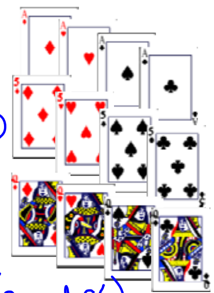
Counting problems flow-chart

Repeats allowed?
- Yes -- Does FCP apply? (verify the unif. criterion)
  - Yes -- Use it!
  - No -- Use table, tree, or rearrange your choices
- No -- Does order matter? (e.g., should ABC and BAC count as 'different' outcomes?)
  - Yes -- Use P(n,r)
  - No -- Use C(n,r) (perhaps in conjunction with the FCP)

Include words like 'at least' or 'at most'?
- Consider using the Complements Principle

Consist of two or more possibilities connected by 'Or'?
- Use the Special (if disjoint) or General Additive Principle.

Poker Hands (see p. 654 for examples):



Practice Questions:

1. How many ways can you select a set of three Aces from a deck of playing cards? $C(4,3) = \frac{4\cdot\cancel{3}\cdot\cancel{2}}{\cancel{3}\cdot\cancel{2}\cdot 1} = 4$

Also: $C(4,1) = 4$

2. My son's birthday is Dec. 9, and his favorite color is red. How many ways could he choose either two 9s or two red cards from a deck?

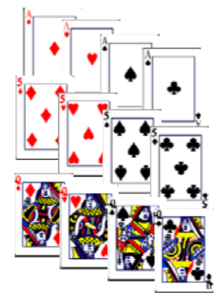
$$n(2\ 9\text{'s}) + n(2\ \text{red}) - n(2\ \text{red}\ 9\text{'s})$$

$$= C(4,2) + C(26,2) - C(2,2)$$

$$= \frac{4\cdot 3}{2!} + \frac{26\cdot 25}{2!} - \frac{2\cdot 1}{2!}$$

$$= 6 + 325 - 1 = 330 \text{ ways}$$

Poker Hands (see p. 654 for examples):



Practice Questions:

3. How many ways are there to select any three cards of the same face value from the deck? (e.g. three 5s or three Ks, etc.)

choose face value	pick 3 of those cards	
13	× C(4,3)	= 13 × 4 = 52

Two pair...

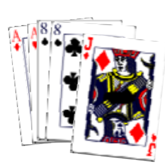
4. In poker, "two pair" is a five-card hand that contains two cards of one face value, two cards of another face value, and a fifth card of a third face value. Show that there are 123,522 possible "two pair" hands.

Break the task into the following choices: why?

1. Select the face values of the two different pairs. $C(13,2)$
2. Select the two cards that make up the first pair. $C(4,2)$
3. Select the two cards that make up the second pair. $C(4,2)$
4. Select the remaining card.

↳ it cannot be one of the face-values you picked in step 1...

$\Rightarrow C(44,1) = 44$



Counting numbers: Find the number of four-digit counting numbers that contain at least one zero under the following conditions:

(a) repeated digits are allowed

(b) repeated digits are not allowed

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(a) $n(\text{at least one zero}) = n(\text{all codes}) - n(\text{codes w/ no zeros})$

$= (9\times 10\times 10\times 10) - (9\times 9\times 9\times 9) = \ldots$

1st digit is not 0.

(b) $n(\text{at least one zero}) = n(\text{all codes}) - n(\text{codes w/ no zeros})$

$= (9\times 9\times 8\times 7) - (9\times 8\times 7\times 6) = \ldots$

not zero

no repeats, but 0 is ok now

still no repeats

no repeats, cannot choose zero for any digit

Counting on Frank...

Frank wants to find out how many ways a 5 digit number can be formed using the digits from the set {1, 2, 3, 4, 5, 6, 7, 8, 9} if he requires that one digit is used twice and no other digit is repeated?

For example, Frank would accept the numbers 3**4**6**4**8 and 245**11**, but he would not accept the numbers 34646 and 34464. How many numbers like this are possible? Show your reasoning.

Find a sequence of choices that would permit the use of the fundamental counting principle.

(Then use it to solve the problem) _ _ _ _ _

Choose one digit to repeat.	choose 2 places to put it	fill in the 3 remaining spots
9 ways	$C(5,2)$	$P(8,3)$
	(just choose 2 of the spots; order of selection is not important)	(order of these 3 digits matters; do not choose the one selected to repeat in step 1)

Multiply to get final answer.