Basics of Probability (Sec. 12.1)

Present Monday: #17, 21, 27

9:55: Molly S Amy S Jared T Briana T Rachel W Jena W
11:00: Kelly S Jeffrey S Kayla T Kelly T Calli V Molly W

Homework:

12.1 #3-11 odd; 12, 15, 17, 18, 39, 40, 42, 43, 49, 51-53, 63

Present Tuesday: 12.1 #39, 52, and 63

9:55 Rebekah W Lucas A Arie B Kelly B Leigha B Alexis B
11:00 Tyson Y Colin A Janessa B Micky B Brittany E Lauren E

(Quick quiz tomorrow on Ch. 11.5 or Ch. 11 'Test')

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☆ Helpers still needed for Emerson Family Math Night on Feb. 24th. Contact Dr. Kosiak (kosiak.jenn@uwlax.edu) or me to sign up!

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8) $\{0,1,2\}$ no repeats of odds

stuck

$\underset{(2\text{ or }1)}{2}$ $\cancel{X}$ _

Two cases:

$\underset{\text{choose 1}}{1} \cdot 2 \cdot 2$ → 4

or $\underset{\text{choose "2"}}{1}$ $\cancel{3}$ $\underset{2\text{ or }3}{}$ stuck again!

Break into two more subcases:

$\underset{\text{choose "2"}}{1} \cdot \underset{\text{choose "1"}}{1} \cdot 2$ → 2 or $\underset{\text{choose 2}}{1} \cdot \underset{\text{choose 0 or 2}}{2} \cdot 3$ → 6

4+2+6 = 12 total

Note that a systematic list isn't terribly difficult here and may be a better way to approach this problem!

100 102 120 122
200 201 202
212 210
220 221 222

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18) 12 players, choose 3 or more.

Two options: (1) count directly, or (2) count the complement and subtract from the total number of subsets.

① $C(12,3) + C(12,4) + C(12,5) + \vdots + C(12,12)$

② $2^{12} - (C(12,0) + C(12,1) + C(12,2))$

why?

Fact: The total number of subsets of an n-item set is 2^n.

Why? Each of the n-items either is or is not in any given subset. So 2 choices per item: 2^n total ways to choose a subset.

Ex: $\{A,B,C\}$

We can describe subsets in terms of whether or not each element is present in the given subset. Two examples follow:

$\{A,C\} \rightarrow \underset{A}{y}\ \underset{B}{n}\ \underset{C}{y}$

$\{\} \rightarrow \underset{A}{n}\ \underset{B}{n}\ \underset{C}{n}$

$\vdots$

There are $2 \cdot 2 \cdot 2 = 2^3$ such descriptions of subsets, and hence 2^3 subsets of any 3-element set.

Quick Counting Review ... and Warm-up

There are 14 juniors and 23 seniors in the Service Club. The club is to send 4 representatives to the State Conference.

1. How many different ways are there to select a group of 4 students to attend the conference? $C(37,4) = 66{,}045$

2. How many ways are there to send two juniors and two seniors to the conference? $C(14,2) \cdot C(23,2) = 23{,}023$

Follow-up: If the students are *randomly* selected, what is the probability that two juniors and two seniors are chosen to attend the conference?

$$P(2\text{ juniors \& }2\text{ seniors}) = \frac{n(2\text{ juniors \& }2\text{ seniors})}{n(\text{any 4 individuals})} = \frac{23{,}023}{66{,}045} \approx 35\%$$

Basic Probability Terminology

- **Experiment 1:** Roll a pair of dice to form a 2-digit number.
  - An **experiment** is any observation or measurement of a random phenomena.
  - **Outcomes** are the possible results of an experiment.
  - The **sample space, S,** is the set of all possible outcomes of an experiment.

S = {11, 12, 13, 14, 15, 16,
21, 22, ... , 26
⋮
61, 62, ... , 66}

- Let E be the event that the 2-digit number is a multiple of 3.
  - * An **event** is a set of (zero, one, or more) outcomes.
  - * It is always subset of the sample space.

E = {12, 15, 21, 24, 33, 36, 42, 45, 51, 54, 63, 66}

Basic Probability Terminology

If two (or more) events have the same chance of occurring, then the events are said to be **equally likely**.

A pair of thought experiments:

1. If a fair coin is tossed end over end and lands on a hard surface, the sample space S is {Heads, Tails}.

   Are the outcomes 'heads' and 'tails' equally likely?

   yes

2. If a gumdrop is tossed end over end and lands on a hard surface, the sample space S is {Lands sideways, Lands upright}

   Are the outcomes 'Lands sideways" and "Lands upright" equally likely? probably not...

Calculating Probabilities

* A **probability** represents the chance of a given event happening; it is a number between 0 and 1, with 1 representing a 100% likelihood that the event will occur.

* If all outcomes in a sample space are equally likely, then the **theoretical probability** of an event E is given by:

$$P(E) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$$

Calculate the Probability:

Four friends, Jon, Kyle, Lori, and Marge, are to be seated in four adjacent seats at a theater. What is the probability that Lori and Marge will be seated next to each other if the seats are assigned randomly?

Theoretical Probability = (# successes) / (total # in sample space)

choose 2 seats	place Lori & Marge	place others
3	2!	2!

$= \frac{12}{4!} = \frac{12}{24} = 50\%$

What are the Odds?

* An **'odds ratio'** is another way to express the likelihood of an event occurring.

* If the probability of a given event is expressed as *a/n*, then the **odds in favor** of E are "*a* to (*n*-*a*)"... in other words, the odds are expressed as "favorable outcomes to unfavorable outcomes".

(typically, the odds ratio is expressed in reduced form, much like you would do for fractions)

Example: Suppose a weather forecaster states that there is a 25% chance of snow today.

Then the **odds in favor** of snow are: 1 : 3, or 1 to 3

Then the **odds against** snow are: 3 : 1, or 3 to 1.

$P = 25\% = .25 = \frac{1}{4}$

What are the Odds?

Ten poker chips numbered consecutively 1 through 10 are placed in a jar. Two chips are drawn at random.

What is the probability of drawing two chips, without replacement, whose sum is 6? What are the odds?

$$P = \frac{n(\text{sum of } 6)}{n(\text{sample space})} = \frac{4}{P(10,2)} = \frac{4}{90}$$

assumed order matters

numerator:

1+5, 2+4, $\cancel{3+3}$, 4+2, 5+1

$$P = \frac{2}{C(10,2)} = \frac{2}{45}$$

assumed order does not matter