Law of Large Numbers

Present Tuesday: 12.1 #39, 52, and 63

9:55 Rebekah W Lucas A Arie B Kelly B Leigha B Alexis B
11:00 Tyson Y Colin A Janessa B Micky B Brittany E Lauren E

Homework: 12.1 #24-26, 33-35, 59, 60

Present Wednesday: 26, 59, 60

9:55 Ashley B Mae B Aspyn B Abby B Kelly Bu Timothy C
11:00 Katie G Courney Ha Sarah H Tyler H Melissa H Samantha H

Quiz today after presentations.

name tags please :)

After reviewing the correct solution (below), write your score on the back of your quiz.

0 = no progress at all; just rewrote problem
0.5 = false start, not based on relevant principles
1 = false start, but sustained effort with some relevant principles
1.5 = significant mistake(s), or significant misunderstanding(s)
2 = mistake near the end or could not finish; also excessive reliance on calculator or 'brute force' methods
2.5 = trivial mistake (e.g. arithmetic error), but work is mostly correct
3 = correct answer and work

If truck license plates in Michigan consist of four digits followed by two letters, how many different license plates could be created that have at least one letter or one digit repeated? (Suggestion: Use the complements principle of counting).

(The complement of "at least one repeat" is "no repeats.")

① # of plates with no repeats is: $P(10,4) \cdot P(26,2) = 5,040 \cdot 650 = 3,276,000.$

② The total # of plates is $10^4 \cdot 26^2 = 6,760,000.$

③ So the # of plates with at least one repeat is $6,760,000 - 3,276,000$, or 3,484,000 plates.

When all outcomes in the sample space S are equally likely, we can calculate the **theoretical probability** of any event E as:

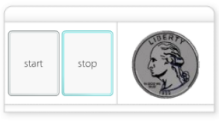
$$P(E) = \frac{n(E)}{n(S)}$$

When we assume all outcomes are equally likely, we are using a **uniform probability model**.

Example: What is the probability of tossing a fair coin three times and getting exactly two heads?

This can be calculated precisely (that is, theoretically) because all outcomes in the sample space are equally likely.

$\frac{n(E)}{n(S)} = \frac{3}{8}$



HHT HTH
TTH THH
HHH THT
TTT HTT

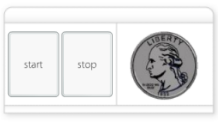
Example: What is the probability of tossing a fair coin two times and getting both a head and a tail? HH, (HT, TH) TT

Like many students, Jimmy doesn't trust theoretical probabilities. He says, "there are 3 outcomes -- two heads, two tails, or one of each -- so the probability should be 1/3."

An experiment can see whether Jimmy is correct.
1. Toss a coin twice and record whether you get "one of each."
2. Repeat the experiment a number of times.
3. Keep track of the empirical probability in a table or chart.

Do you think the evidence from this experiment is strong enough to convince Jimmy that the correct probability is not 1/3?

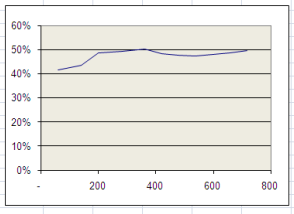
Law of Large Numbers states that the empirical probability tends to approach the theoretical probability as the number of trials increases.



Coin Toss Results (HT or TH)

C12 =SUM(B$2:B12)/SUM(A$2:A12)

	A	B	C	D	E	F
1	Num Trials	Num Successes	Cumulative P(E)	Cumulative Trials		Coin Toss
2	60	25	42%	60		
3	80	36	44%	140		
4	60	36	49%	200		
5	80	41	49%	280		
6	80	43	50%	360		
7	60	22	48%	420		
8	60	26	48%	480		
9	56	24	47%	536		
10	60	33	48%	596		
11	60	32	48%	656		
12	60	36	49%	716		
13	716					



Find the empirical probability that a Hershey's kiss tossed into the air lands on its base.

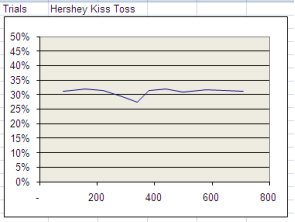
First, make a prediction. Will the probability be:

10%? 25%? 33%? 50%? 67%? 75% 90%?

Hershey Kiss Toss Results

C12 =SUM(B$2:B12)/SUM(A$2:A12)

	A	B	C	D	E	F
1	Num Trials	Num Successes	Cumulative P(E)	Cumulative Trials		Hershey Kiss Toss
2	80	25	31%	80		
3	80	26	32%	160		
4	60	18	31%	220		
5	60	14	30%	280		
6	60	10	27%	340		
7	40	26	31%	380		
8	60	22	32%	440		
9	60	13	31%	500		
10	80	30	32%	580		
11	70	21	32%	650		
12	60	17	31%	710		
13	710					



A pair of dice are rolled. Find the probability that the outcome has at least one five.

$n(E) = n(S) - n(E')$

$= 6^2 - 5^2$

$= 11$

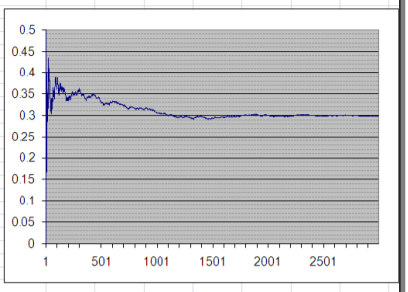
We can calculate the theoretical probability using the complements principle (this assumes all outcomes are equally likely, or that we have a uniform probability model).

$$P(E) = \frac{n(E)}{n(S)} = \frac{11}{6^2} = \frac{11}{36} \approx 30.5\%$$

We can also simulate an experiment like this using computers. This gives an empirical probability estimate.

Two Dice Simulation (At Least 1 Five)

	3075		918	0.298537
red die	blue die	at least 1 five?		
4	1	FALSE	0	
5	3	TRUE	0.5	
1	0	FALSE	0.333333	
6	2	FALSE	0.25	
6	0	FALSE	0.2	
4	3	FALSE	0.166667	
5	4	TRUE	0.285714	
5	0	TRUE	0.375	
4	4	FALSE	0.333333	
6	5	TRUE	0.4	
1	1	FALSE	0.363636	
1	3	FALSE	0.333333	
6	2	FALSE	0.307692	
1	2	FALSE	0.285714	
3	5	TRUE	0.333333	
6	3	FALSE	0.3125	
5	2	TRUE	0.352941	
6	4	FALSE	0.333333	
4	3	FALSE	0.315789	
0	5	TRUE	0.35	
1	5	TRUE	0.380952	
5	4	TRUE	0.409091	



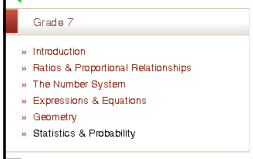
Project 1: Counting & Probability

The Common Core Standards for Mathematics

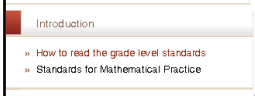
COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Two types of standards:

* Content (listed by grade-level for K-8; by subject in HS)
* Process ("mathematical practice standards"; common across all grade levels)



See Grade 7 Standards



See Mathematical Practice Standards (in the CCSM introduction; also in the overview / introduction of *every* grade level)

Resources: NCTM's *Navigations* Series (Navigating through Data Analysis and Probability in Grades PK-2; 3-5; 6-8.)