Probabilities with "AND"

Present today: #1, 15, 33

9:55 Kaitlyn Hi., Amanda, Grace, Molly, Azjja, Katie Kl.
11:00 Gen Mc., Danielle, Steven, Emily, Jordan, Kyle Po.

HW 12.3 #1-3, 13, 17, 19, 23-27, 33-34, 35-36
Present tomorrow: #19, 25, 33

9:55 Megan Mi., Katie, Alyssa, Nicole, Rachel, Hannah Ro.
11:00 Rebecca Pr., Emma, Jamie, Erin, Jamie, Victoria Sc.

Warm-up (Use any method):

1. A card is drawn from a deck and replaced before a second card is drawn. Find the probability that both cards are jacks.

2. Two cards are drawn from a deck. Find the probability that both cards are jacks.

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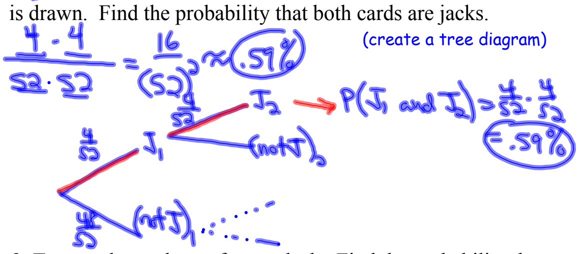
(create a tree diagram)



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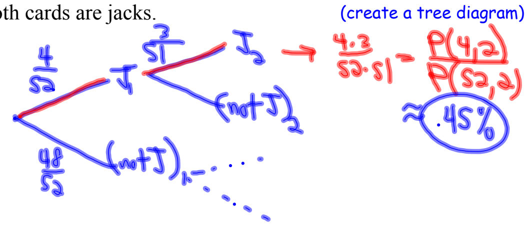
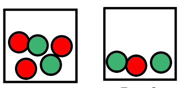


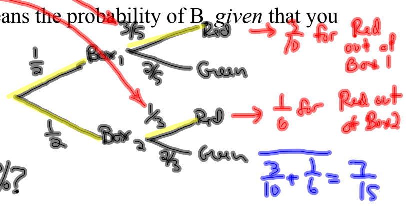
Lost your marbles?

Example: Box 1 contains 3 red and 2 green marbles. Box 2 contains 1 red and 2 green marbles. If a box is chosen at random and a marble is drawn, find the probability that the marble is red.

Tip: Create a tree diagram

Notation: P(B | A) means the probability of B, *given* that you know A has occurred.





What if P(Box₁) = 90%?

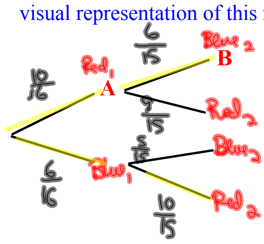
Incorrect: $\frac{4}{8}$ (Dump boxes together)

Incorrect: $\frac{3}{5} + \frac{1}{3} = \frac{14}{15}$ → add probabilities... (either Box₁ or Box₂)

General Multiplicative Rule (AND)

If A and B are any two events, then
P(A and B) = P(A) * P(B | A)

As we have seen, a tree diagram can be used to give a visual representation of this rule.



Example: You are told to pick two marbles from a jar that contains 6 blue and 10 red marbles.

Find the probability of picking both a red and blue marble (i.e., one of each).

$P(\text{one of each}) = P(R_1 \text{ and } B_2 \text{ OR } B_1 \text{ and } R_2)$

$= P(R_1 \text{ and } B_2) + P(B_1 \text{ and } R_2)$

$= P(R_1) \cdot P(B_2 | R_1) + P(B_1) \cdot P(R_2 | B_1)$

$= \frac{10}{16} \cdot \frac{6}{15} + \frac{6}{16} \cdot \frac{10}{15}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Some New Notation

(Conditional Probabilities)

Example: Two dice are rolled and the sum is calculated. Let A be the event that the sum is even, and let B be the event that the sum is prime. Then...

P(A) = P(B | A) =

P(B) = P(A | B) =

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Caution: How is P(A and B) different from P(A | B)? from P(B | A)?

Some New Notation

(Conditional Probabilities)

Example: Two dice are rolled and the sum is calculated. Let A be the event that the sum is even, and let B be the event that the sum is prime. Then...

P(A) = $\frac{18}{36}$ P(B | A) = $\frac{1}{18}$

P(B) = $\frac{15}{36}$ P(A | B) = $\frac{1}{15}$ = $\frac{\text{\# of even prime}}{\text{\# prime}}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
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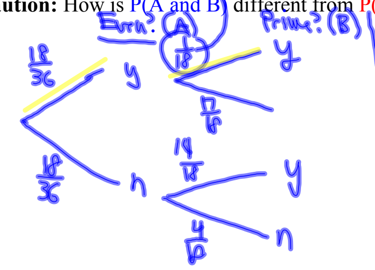
6	7	8	9	10	11	12

Caution: How is P(A and B) different from P(A | B)? from P(B | A)?



P(A and B) = $\frac{1}{36}$

P(A)·P(B | A) = $\frac{18}{36} \cdot \frac{1}{18} = \frac{1}{36}$

Some New Notation

(Conditional Probabilities)

Example: Two dice are rolled and the sum is calculated. Let A be the event that the sum is even, and let B be the event that the sum is prime.

This conditional probability notation is used to define what it means for two events to be **independent**:

Events A and B are **independent** if P(B | A) = P(B)

Are the events A and B described above *independent*?

P(B | A) = $\frac{1}{18} \overset{?}{\neq} \frac{15}{36}$

Four of a kind?

Example: In a game of chance similar to Yahtzee, you can roll four dice up to three times in an effort to make all four dice match. You can roll any or all of the dice on each turn.

Suppose you have rolled the dice on your first turn and they came up with a pair of 2's. What is the probability of making all four dice match in your next two turns?

Tip: Create a tree diagram

What role does *independence* play in this problem?

Birthday Twins?

In any group of three individuals, find the probability that two of them have the same birthday.

P(at least two are the same)
= 1 - P(all are different)
= 1 - P(365,3)/(365^3)
= 1 - 364*363 / 365*365
≈ 1 - .992
≈ 0.008

P(A and B) = P(A) * P(B | A)

For Group Discussion

1. Based on the data shown in the table, what are the odds in favor of a duplication in a group of 30 people?
2. Estimate from the table the least number of people for which the probability of duplication is at least 1/2.
3. How small a group is required for the probability of a duplication to be *exactly* 0?
4. How large a group is required for the probability of a duplication to be *exactly* 1?

Number of People	Probability of at Least One Duplication	Number of People	Probability of at Least One Duplication	Number of People	Probability of at Least One Duplication
2	.003	19	.379	36	.832
3	.008	20	.411	37	.849
4	.016	21	.444	38	.864
5	.027	22	.476	39	.878
6	.040	23	.507	40	.891
7	.056	24	.538	41	.903
8	.074	25	.569	42	.914
9	.095	26	.598	43	.924
10	.117	27	.627	44	.933
11	.141	28	.654	45	.941
12	.167	29	.681	46	.948
13	.194	30	.706	47	.955
14	.223	31	.730	48	.961
15	.253	32	.753	49	.966
16	.284	33	.775	50	.970
17	.315	34	.795		
18	.347	35	.814		