

Present Thursday: #15, 19, 25

9:55 Briana Tor., Rachel, Jena, Rebekah Wol., Luke, Arie
 11:00 Kelly Tom., Calli, Molly, Tyson You., Colin, Janessa

Homework: Sec. 8.7 #67, 68, 69, 70, 72, 77, 85

* Note:

The "tables" provided in your book on #67, 68, and 72 will NOT be provided on exams or quizzes. They are not what we mean when we refer to *multiple representations*. Rather, the tables the book provides are a scaffold intended to help you set up the problem as you learn. With practice, this scaffold should eventually no longer be needed.

It may, however, be helpful to use a *guess & check table* to set up the problem. I'll show you some examples of that in today's notes. The use of a guess & check table is grounded in the standards and is an excellent problem solving tool.

Present Monday: #69, 77, 85

9:55 Kelly Bei., Leigha, Alexis, Ashley, Mae, Aspyn Bre.
 11:00 Mykki Bor., Brittany, Lauren, Katie, Courtney, Sarah Had.

Reminder: Algebra Projects are Due on 4/21 (two weeks from today)

After reviewing the correct solution (below), write your score on the back of your quiz.

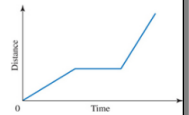
- 0 = no progress at all; just rewrote problem
- 0.5 = false start, not based on relevant principles
- 1 = false start, but sustained effort with some relevant principles
- 1.5 = significant mistake(s), or significant misunderstanding(s)
- 2 = mistake near the end or could not finish; also excessive reliance on calculator or 'brute force' methods
- 2.5 = trivial mistake (e.g. arithmetic error), but work is mostly correct
- 3 = correct answer and work

The following student responses reflect a common misconception (held by students at both the elementary and high school levels) about the meaning of the distance-time graph shown:

Student 1: "First I started walking up a hill. Then the hill flattened out. Then there was another, steeper hill that I walked up."

Student 2: "I rode my bike down a street. Then I turned a little to the right and rode for a while. Then I turned left and rode a long way."

This is called the "graph as a picture" misconception. Write a simple story that reflects the **correct interpretation** of the distance-time graph shown.



Answers may vary, but should go something like this:

- a) traveled away from "home" for a while at a constant rate of speed,
- b) then stopped (for about the same amount of time),
- c) then traveled away from home again at about twice the rate of speed.

Example: The last part of the triathlon is a 10K (10 kilometers, or 10,000 meters) run. When competitor Aña starts running this last part, she is 600 m behind competitor Bea. But Aña can run faster than Bea can. Aña can run (on average) 225 m/min, and Bea can run (on average) 200 m/min. Who wins, Aña or Bea? If Aña wins, tell when she catches up with Bea. If Bea wins, tell how far behind Aña is when Bea finishes.



Example: The last part of the triathlon is a 10K (10 kilometers, or 10,000 meters) run. When competitor Aña starts running this last part, she is 600 m behind competitor Bea. But Aña can run faster than Bea can. Aña can run (on average) 225 m/min, and Bea can run (on average) 200 m/min. Who wins, Aña or Bea? If Aña wins, tell when she catches up with Bea. If Bea wins, tell how far behind Aña is when Bea finishes.

Strategy: Make a Table

Version 1

Time (min)	Aña's position (m)	Bea's position (m)
0	0	600
1	225	800
2	450	1000
3	675	1200
4	900	1400
5	1125	1600
⋮	⋮	⋮

Version 2

Time (min)	Aña's position (m)	Bea's position (m)
0	0	600
5	1125	1600
10	2250	2600
15	3375	3600
20	4500	4600
25	5625	5600

24 | 5400 | 5400

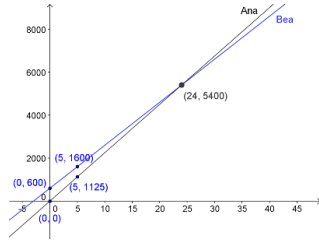
About $\frac{1}{2}$ way thru, Aña catches up. So she will win the race.

Example: The last part of the triathlon is a 10K (10 kilometers, or 10,000 meters) run. When competitor Aña starts running this last part, she is 600 m behind competitor Bea. But Aña can run faster than Bea can. Aña can run (on average) 225 m/min, and Bea can run (on average) 200 m/min. Who wins, Aña or Bea? If Aña wins, tell when she catches up with Bea. If Bea wins, tell how far behind Aña is when Bea finishes.

Strategy: Use a Graph

* Uses the first few rows of the table

* May not give a precise answer (but shows the answer is ~25 min).



Example: The last part of the triathlon is a 10K (10 kilometers, or 10,000 meters) run. When competitor Aña starts running this last part, she is 600 m behind competitor Bea. But Aña can run faster than Bea can. Aña can run (on average) 225 m/min, and Bea can run (on average) 200 m/min. Who wins, Aña or Bea? If Aña wins, tell when she catches up with Bea. If Bea wins, tell how far behind Aña is when Bea finishes.

Strategy: Write Equation(s) / Use Algebra

A table or graph can lead us to an equation.

Time (min)	Aña's position (m)	Bea's position (m)
0	0	600
1	225	800
2	450	1000
⋮	⋮	⋮

Handwritten notes and equations:

- Blue arrows pointing from the table to the equations: 225 from the Aña's position column and 200 from the Bea's position column.
- Equation for Aña: t (vertical line) | $225t$
- Equation for Bea: $200t + 600$
- Intersection point: $t = 24$

Example: The last part of the triathlon is a 10K (10 kilometers, or 10,000 meters) run. When competitor Aña starts running this last part, she is 600 m behind competitor Bea. But Aña can run faster than Bea can. Aña can run (on average) 225 m/min, and Bea can run (on average) 200 m/min. Who wins, Aña or Bea? If Aña wins, tell when she catches up with Bea. If Bea wins, tell how far behind Aña is when Bea finishes.

Strategy: Use Reasoning

Compare the rates. How long will it take to overcome the gap?

Handwritten notes:

- difference in rates = 25 m/min.
- It takes 24 min to catch up (600 ÷ 25).

Applications of Algebra: Problem Solving Strategy

1. Define your variables; be specific!
2. Write equations; a table or initial guess can help!
3. Solve the equations.
4. State the answer.
5. Is answer reasonable?

More Applications of Algebra:

A note about concentration of a mixture: We say an acid-water mixture is "15% acid" if the ratio of pure acid to pure water in the mixture is 15 to 85. That is, it contains 15 parts pure acid and 85 parts pure water.

Acid Mixture Warm-up: How many liters of pure acid and how many liters of pure water were mixed to create 60 liters of:

- a) 50% acid? 30 L acid, 30 L water
- b) 10% acid? 6 L acid, 54 L water
- c) 5% acid? 3 L acid, 57 L water

80 L acid, 20% concentration.

More Applications of Algebra:

Acid Mixture: How many liters each of 15% acid and 33% acid should be mixed to obtain 40 liters of 21% acid?

1. Define your variables:

Let x = # liters of 15% acid

Let y = # liters of 33% acid

8.4 L acid
31.6 L water

2. Write equations. (Consider using a table or a guess first...)

x	y	pure acid in x	pure acid in y	correct?
30	10	$30(.15) = 4.5$ L	$10(.33) = 3.3$ L	$4.5 + 3.3 = 7.8$ $\neq 8.4$
x	$40 - x$	$x(.15)$	$(40 - x)(.33)$	$? = 8.4$

$x \approx 26.7?$

More Applications of Algebra:

Acid Mixture: How many liters each of 15% acid and 33% acid should be mixed to obtain 40 liters of 21% acid?

- 3. Solve the equations.
- 4. State the answer.
- 5. Ask "Is it reasonable?"

Topic of the day: Applications

Speeds of Trains: A train travels 150 km in the same time that a plane covers 400 km. If the speed of the plane is 20 kph less than 3 times the speed of the train, find both speeds.

1. Define your variables:

Let r = speed of train (in kph)

Let p = speed of plane (in kph)

Guess $t = 50$ kph.
 $p = 130$ kph.
 $(150 - 20)$

2. Write equations.

Let t = common time.
 $150 = r \cdot t \rightarrow t = \frac{150}{r}$
 $400 = p \cdot t \rightarrow t = \frac{400}{p}$

$p = 3r - 20$

3. Solve the equations.

$\frac{150}{r} = \frac{400}{p}$

$\frac{150}{r} = \frac{400}{3r - 20} \Rightarrow 150(3r - 20) = 400r$
 $450r - 3000 = 400r$

$50r = 3000$

$r = 60$ kph.

$p = 3(60) - 20 = 160$

Topic of the day: Applications

Speeds of Trains: A train travels 150 km in the same time that a plane covers 400 km. If the speed of the plane is 20 kph less than 3 times the speed of the train, find both speeds.

4. State the answer.

The train travels at 60 kph, and the plane travels at 160 kph.

5. Is answer reasonable?

Yes, these rates of travel are appropriate speeds for a train and a plane, respectively, so our answers seem reasonable.

Topic of the day: Applications

Boat and Current: In his motorboat, Nguyen travels upstream at top speed a distance of 36 miles in two hours. Returning, he finds that the trip downstream, still at top speed, takes only 1.5 hours. Find the speed of Nguyen's boat and the speed of the current.

1. Define your variables:

Let b = speed of the boat (in mph)

Let c = speed of the current (in mph)

2. Write equations.

3. Solve the equations.

Topic of the day: Applications

Boat and Current: In his motorboat, Nguyen travels upstream at top speed a distance of 36 miles in two hours. Returning, he finds that the trip downstream, still at top speed, takes only 1.5 hours. Find the speed of Nguyen's boat and the speed of the current.

4. State the answer.

You should find that the current is 3mph and Nguyen's boat's top speed is 21 mph.

5. Is answer reasonable?