Mth126 Algebra Unit P/review

The following problems include some of the more recent problem types we have worked with. You do not need to do these in order – within your group, work on those you think you will have the most difficulty with, and ask questions!

Optimization problem, from purplemath.com:

1. You need to buy some filing cabinets. You know that Cabinet X costs $10 per unit, requires six square feet of floor space, and holds eight cubic feet of files. Cabinet Y costs $20 per unit, requires eight square feet of floor space, and holds 10 cubic feet of files. You have been given $140 for this purchase, though you don't have to spend that much. The office has room for no more than 72 square feet of cabinets. How many of which model should you buy, in order to maximize storage volume? *(Ans: x=12, y = 0; it barely beats out x=8, y=3; see graph below)*

Let x = # of Cabinet X purchased, and
y = # of Cabinet Y purchased.

Constraints are:

price: $10x + 20y \le 140$ → $x + 2y \le 14 \Rightarrow x = 14 - 2y$

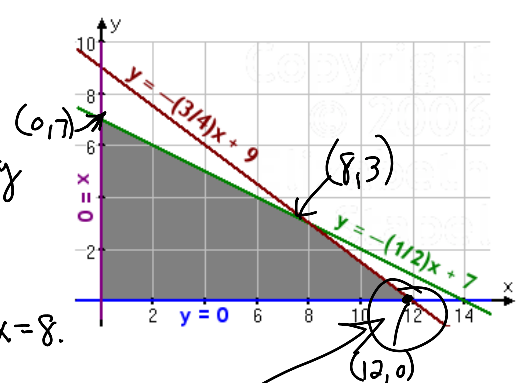
floor: $6x + 8y \le 72$

Also $x \ge 0$, $y \ge 0$.

Intersection:

$\rightarrow 6(14-2y) + 8y = 72$
$\Rightarrow 84 - 12y + 8y = 72$
$\Rightarrow -4y = -12 \Rightarrow y = 3 \Rightarrow x = 8.$

This system (along with the first two constraints) graphs as:



Objective function: (storage): $8x + 10y = \max?$

x	y	8x + 10y
0	7	70 cu.ft
12	0	96 cu.ft ← Best!
8	3	94 cu.ft

Solve the following inequalities.

2. $\left|\frac{3}{2}x - 7\right| \ge 5$

$\frac{3}{2}x - 7 \le -5$ or $\frac{3}{2}x - 7 \ge 5$

$\cancel{\frac{2}{3}}\cdot\cancel{\frac{3}{2}}x \le 2 \cdot \frac{2}{3}$ or $\cancel{\frac{2}{3}}\cdot\cancel{\frac{3}{2}}x \ge 12 \cdot \frac{2}{3}$

$x \le 4/3$ or $x \ge 8$

$\rightarrow (-\infty, \frac{4}{3}] \cup [8, \infty)$

$\frac{4}{3}$ $\quad$ 8

3. $|2x + 5| < -4$

No solutions! ($|2x+5| \ge 0$ for any x, so it will not be less than -4... ever!)

4. $|2x + 3| < 5$

$-5 < 2x + 3 < 5$
$-8 < 2x < 2 \Rightarrow -4 < x < 1$

$(-4, 1)$

-4 $\quad$ 1

Factor. First use factor by grouping, then use common factor quotient (CFQ).

5. $6x^2 - 5x - 4$

$ac = -24$, $b = -5$ $\}$ $(-8)(3)$ $\}$ $6x^2 - 8x + 3x - 4$ $= 2x(3x-4) + (3x-4)$ $= (2x+1)(3x-4)$

$\dfrac{(6x-8)(6x+3)}{6} = \dfrac{\cancel{2}(3x-4)\cancel{3}(2x+1)}{\cancel{6}} = (3x-4)(2x+1)$

Complete the square and solve:

6. $4x^2 + 24x - 12 = 0$

$\Rightarrow x^2 + 6x - 2 = 0$ (divided by 4)

$\Rightarrow x^2 + 6x + \underline{9} = 2 + \underline{9}$

$\Rightarrow (x+3)^2 = 11 \Rightarrow x = -3 \pm \sqrt{11}$

Complete the square to find the vertex, and then find the x- and y-intercepts.

7. $g(x) = -x^2 + 8x + 2$

$= (-1)(x^2 - 8x + \underline{16}) + 2 + \underline{16}$

added $(-1)(16)$ — $(-4)^2$ — so add $+16$ here to compensate

$= -1(x-4)^2 + 18 \Rightarrow$ vertex $= (4, 18)$.

x-intercepts:

$-(x-4)^2 + 18 = 0$

$\Rightarrow -(x-4)^2 = -18$

$\Rightarrow (x-4)^2 = 18$

$\Rightarrow x = 4 \pm \sqrt{18} = 4 \pm 3\sqrt{2}$

y-int: $g(0) = 2$

Solve the following optimization problem (#58 in Sec. 8.5):

8. For a trip to a resort, a charter bus company charges a base fare of \$48 per person, but adds \$2 per person for each unsold seat on the bus. The bus has 42 seats.
   a. What is the total revenue if there are 0 unsold seats?
   b. What is the total revenue if there are 3 unsold seats? (Hint: the fare will be \$54 per person).
   c. Write a function for the total revenue if there are *x* unsold seats.

9. (Refer to previous problem). How many passengers on the bus would bring in the greatest revenue to the bus company? What is this maximum revenue?

(ans: 33 passengers gives \$2,178 in revenue)

8) a) $\$48(42) = \$2{,}016$

b) $(\$48 + \$6)(39) = \$2{,}160$

c) $(48 + 2x)(42 - x) = R(x)$, or $(48)(42-x) + 2x(42-x)$.

9) $R(x) = 2016 - 48x + 84x - 2x^2 = -2x^2 + 36x + 2016$

axis of symmetry: $x = \dfrac{-36}{2(-2)} = \dfrac{-36}{-4} = 9$ empty seats at the axis (and hence the vertex).

$R(9) = \$2{,}178$