

Teaching for Understanding and Its Impact on Learning in Algebra

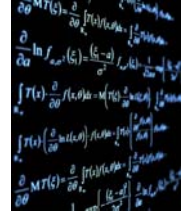
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Handouts and slides available after the conference at:
www.uwlax.edu/faculty/hasenbank/talks/nctm09

Overview

- Background & NCTM Focus Connection
- Transforming "Traditional Tasks" into "Understanding Tasks"
- Sharing Activities and Assessments
- Some Results
 - Pre-post test comparisons
 - A closer look: skill \Leftrightarrow understanding?
- Conclusions



http://www.uofaweb.uaberta.ca/compneur/olab/images/math_400.jpg

First, Your Examples...

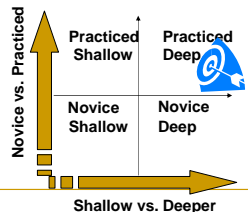
- Write down a specific problem that involves a *math procedure* that you taught in the last week.
- How did your students do? What will they remember in 6 months?
 - How does that compare with what an "expert" would know and do?
 - How can we bridge that gap?

Now, Our Examples...

- Solve:
 1. $2x - 1 = 6$
 2. $|x - 3| < 5$
 3. $x^2 + 3x - 4 = 0$
 4. $2x + 3y < 6$
 5. $3x + 4y = 14$, and $y = -(5/2)x$
- Tasks: Vehicles for Procedures and Concepts

Dimensions of Knowledge

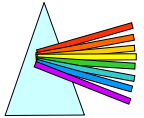
- Type of Knowledge
 - Concepts, Procedures, Facts
- Other Dimensions:
 - Shallow vs. Deep
 - Novice vs. Practiced



Another Look...

- "Procedural fluency includes not only knowing how to carry out procedures, but also understanding why they work, how they might be used, and how the results should be interpreted...." (NCTM Focus in High School Math: Reasoning & Sense making, 2009)
- What questions could we ask to enrich these tasks to promote deeper knowledge?
 - Solve: $2x - 1 = 6$
 - Solve: $|x - 3| < 5$
 - Solve: $x^2 + 3x - 4 = 0$
 - Solve: $2x + 3y < 6$
 - Solve: $3x + 4y = 14$ and $y = -(5/2)x$

Framework for Procedural Understanding (or, Eight “Reasoning Habits”)



- 1a) What is the **goal** of the procedure?
- 1b) What sort of answer should I **expect**?
- 2a) How do I **carry out** the procedure?
- 2b) What **other procedures** could I use?
- 3) **Why** does the procedure work?
- 4) How can I **verify** my answer?
- 5) When is this the **“best”** procedure to use?
- 6) **What else** can I use this procedure to do?

View the following problems through the lens of the Framework:

Solve: $2x - 1 = 6$

Solve: $|x - 3| < 5$

Solve: $x^2 + 3x - 4 = 0$

Adapted from NCTM's Navigating through Algebra in Grades 9-12

Transforming Our Questions

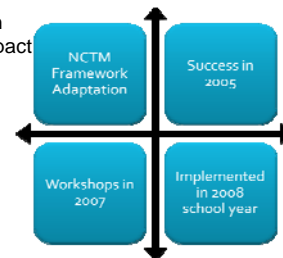
- What's the next step?
- Solve this system of equations by elimination.
- Let's check our answer by “plugging it back in” (by substitution).
- What could we do next, and why?
- Write a system of equations for which the answer is (4, 5).
- How would we know if we made a mistake?
- Is our answer reasonable?

Benefits of Understanding

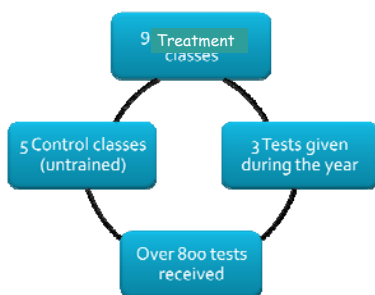
- Reasoning & sense making help students develop connections..., increasing their likelihood of understanding and retaining the new information. (NCTM Focus in High School Math: Reasoning & Sense Making, 2009)
- Knowledge that is understood is more robust and can be used flexibly within and across different contexts. (Carpenter & Lehrer, 1999; Hiebert & Carpenter, 1992; Van Hiele, 1986)
- Students should use procedures “flexibly, accurately, efficiently, and appropriately” (National Research Council, 2001)

Impact on Learning

When teachers focus on understanding, what impact does it have on student learning?



Study Design



Sample Assessment Tasks

(How would your students respond?)

- 3.1) What is the value of the expression $(x+1)(x+2)$ when $x = -4$?
- 3.2) Could $(x+1)(x+2)$ ever be a negative number? Explain.
- 3.3) Solve this equation for x . Show your steps. $5x - 6 = 4x + 3$
- 3.4) Describe two ways you could check your answer to the previous question.

Sample Assessment Tasks

(How would your students respond?)

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?
- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

Quantifying Understanding

- Task-specific rubrics were created for evaluating the degree of understanding:

0 = no understanding
1 = low understanding
2 = moderate understanding
3 = high understanding

Item 3.15

(Procedural Task)

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?

0435

15. If $3x + 4y = 14$ and $y = -\frac{5}{2}x$, what is the value of x ?

$$3x + 4(-2.5) = 14$$
$$3x + 10 = 14$$
$$-10 \quad -10$$
$$3x = 4$$
$$x = \frac{4}{3} = 1\frac{1}{3} = 1.\bar{3}$$

Item 3.15

(Procedural Task)

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?

0126

15. If $3x + 4y = 14$ and $y = -\frac{5}{2}x$, what is the value of x ?

$$3x + 4\left(-\frac{5}{2}\right) = 4$$
$$3x - \frac{20}{2}x = 4$$
$$3x - 10x = 4$$
$$\frac{-7x}{-7} = \frac{4}{-7}$$
$$x = -\frac{4}{7}$$

Rubric for Item 3.15

(Procedural Task)

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?
- 0: Blank or no honest effort (e.g. "I don't know (IDK.)")
 - 1: Two or more mistakes present OR an incomplete, illogical, confusing, or way off track response.
 - 2: Only one mistake with an attempted answer.
 - 3: Correct answer is provided.

Item 3.16

(Understanding Task)

- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

0435

16. What does the answer to the previous problem tell you about the graphs of the two equations?

That they are on graphs

0630

16. What does the answer to the previous problem tell you about the graphs of the two equations?

This shows that the line will be negative.

(Note: 0630 correctly answered $x = -2$ on #3.15)

Item 3.16

(Understanding Task)

- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?
- What would a student have to show you to earn a 2 or a 3 on this task?
- Can you find any 2's & 3's?

2 = moderate understanding
3 = high understanding

Rubric for Item 3.16

(Understanding Task)

- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?
 - 0: Blank or no honest effort (e.g. "I don't know (IDK).")
 - 1: Response is vague, illogical, confusing, or way off track.
 - 2: States "they will intersect" OR "it tells you whether they intersect or are parallel."
 - 3: States "they will intersect at $x = -2$ " (or previous problem's x value.)

Sample Responses – Item 3.16

- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

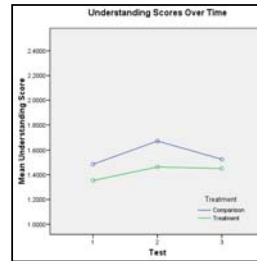
The lines would touch at $(-2, 5)$ if you were to graph them.

Scored: 3
(other)

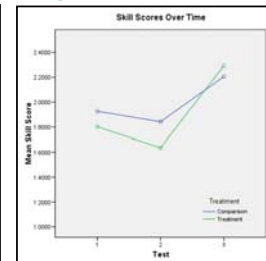
16. What does the answer to the previous problem tell you about the graphs of the two equations?
In less they are parallel they will always intersect.

Scored: 2
(0789)

Results – Understanding & Skill



Gains in understanding?
Test1 → Test3, $p = .413$ (n.s.)



Gains in skill?
Test1 → Test3, $p = .003$

Does Skill Imply Understanding?

(Does Understanding Imply Skill?
Or Are they Independent?)

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?
- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

		q3.16 (understanding)				
		0	1	2	3	Total
q3.15 (skill)	0	40	5	0	0	45
	1	35	73	11	1	120
	2	5	36	3	0	44
	3	8	32	5	3	48
Total		88	146	19	4	257

Does Skill Imply Understanding?

(Does Understanding Imply Skill?
Or Are they Independent?)

- Compare performances on 3.1 and 3.2

1. What is the value of the expression below when $x = -4$?
 $(x+1)(x+2)$
 $(-4+1)(-4+2)$
 $(-3)(-2)$ 6

2. If we could change x to equal something else, could $(x+1)(x+2)$ ever be a negative number? Explain.
No, because the result would either be two negatives, two positives, or a number times 0, so it couldn't be negative.

Does Skill Imply Understanding?

- 3.1) What is the value of the expression $(x+1)(x+2)$ when $x = -4$?
- 3.2) Could $(x+1)(x+2)$ ever be a negative number? Explain.

		q3.2 (understanding)					
		Score	0	1	2	3	Total
q3.1 (skill)	0	9	1	0	0	10	
	1	3	28	3	1	35	
	2	1	10	3	0	14	
	3	2	150	40	6	198	
	Total	15	189	46	7	257	

Does Skill Imply Understanding?

(Does Understanding Imply Skill?
Or Are they Independent?)

- Compare performances on 3.3 and 3.4

3. Solve this equation for x . Show your steps.

$$5x - 6 = 4x + 3$$

$$x = 9$$

4. Describe two ways you could check your answer to the previous question.

One way to check my answer:
you can put what you think x is in the original problem & see if it's right.

Another way to check my answer:
You could go back & recheck your steps.

Does Skill Imply Understanding?

- 3.3) Solve this equation for x . Show your steps. $5x - 6 = 4x + 3$
- 3.4) Describe two ways you could check your answer to the previous question.

		q3.4 (understanding)					
		Score	0	1	2	3	Total
q3.3 (skill)	0	4	1	0	0	5	
	1	7	11	16	6	40	
	2	1	2	5	0	8	
	3	6	14	162	22	204	
	Total	18	28	183	28	257	

Now, Back to You

- (Re)consider the task you wrote on your paper...
- Write a revision or a follow-up that you could use to deepen or enrich the nature of the task.
- (Th)ink-Pair-Share

Conclusion & Discussion

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- Students do not automatically develop (or communicate?) deep knowledge.
- These "habits of mind*" (or "reasoning habits**") should be **modeled during instruction** and **valued during practice & assessment**.
 - Focus is needed on habits such as communication, multiple representations, flexibility and alternate methods.
- What "reasoning habits" are most important for success in your class? In your work? In life?

* Driscoll uses "Habits of Mind"; NCTM's *Focus* uses "Reasoning Habits"