

Promoting Student Understanding of Algebra

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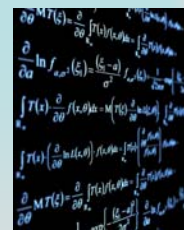
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www.uwlax.edu/faculty/hasenbank/talks/wmc2009

Overview

- Background & Teaching for “Understanding”
- Transforming “Traditional Tasks” into “Understanding Tasks”
- Sharing Activities and Assessments
- Some Results
 - Pre-post test comparisons
 - A closer look: state of knowledge
- Conclusions



http://www.uofaweb.uAlberta.ca/compneur/olab/images/math_400.jpg

First, Some Examples

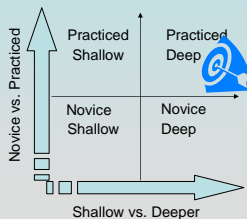
- On your note cards, write a specific question that involves a *math procedure*.
- Answer your question, and consider:
 - What would an “expert” know and do?
 - How does this compare to what your students know and do?
 - How can we bridge that gap?

Our Examples

- Solve:
 1. $2x - 1 = 6$
 2. $|x - 3| < 5$
 3. $x^2 + 3x - 4 = 0$
 4. $2x + 3y < 6$
 5. $3x + 4y = 14$, and $y = -(5/2)x$
- Consider:
 - What would an “expert” know and do?
 - How does this compare to what your students know and do?
 - How can we bridge that gap?

Dimensions of Knowledge

- Type of Knowledge
 - Concepts, Procedures, Facts
- Other Dimensions:
 - Shallow vs. Deep
 - Novice vs. Practiced

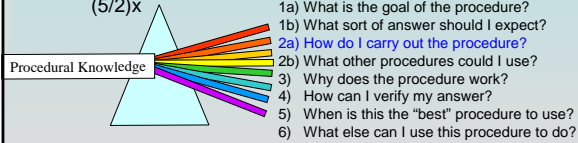


Another Look...

- What questions could we ask to enrich this task to promote deeper knowledge?
 - Solve: $2x - 1 = 6$
 - Solve: $|x - 3| < 5$
 - Solve: $x^2 + 3x - 4 = 0$
 - Solve: $2x + 3y < 6$
 - Solve: $3x + 4y = 14$ and $y = -(5/2)x$

Framework for Procedural Understanding (or, Eight Algebraic Habits of Mind)

Solve: $2x - 1 = 6$
 Solve: $|x - 3| < 5$
 Solve: $x^2 + 3x - 4 = 0$
 Solve: $2x + 3y < 6$
 Solve: $3x + 4y = 14$ and $y = -$
 $(5/2)x$



Adapted from NCTM's Navigating through Algebra in Grades 9-12

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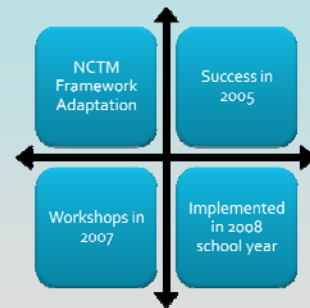
Benefits of Understanding

- If existing knowledge is understood, new knowledge is easier to learn and understand. (Hiebert & Carpenter, 1992)
- Knowledge that is understood is more robust and can be used flexibly within and across different contexts. (Carpenter & Lehrer, 1999; Hiebert & Carpenter, 1992; Van Hiele, 1986)
- Students should use procedures "flexibly, accurately, efficiently, and appropriately" (National Research Council, 2001)

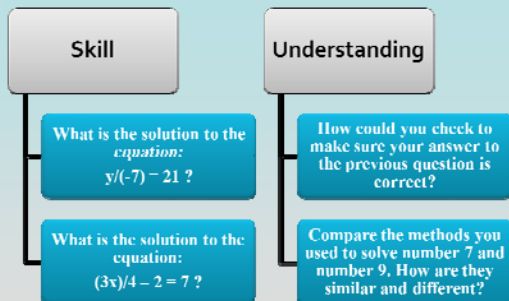
Putting "TfU" to the Test

- When teachers focus on understanding, what impact does it have on student learning?
- Algebra tests given to students in 9 participating classrooms and 5 comparison classrooms.

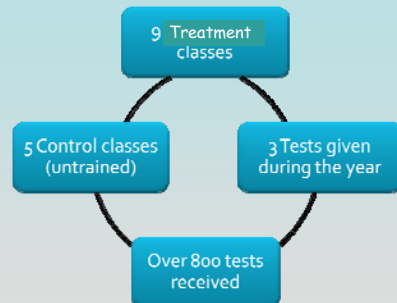
Timeline of Progress



Treatment: Beyond Raw Skill



Our 2007-08 Teaching Experiment



Sample Assessment Tasks

- 3.1) What is the value of the expression $(x+1)(x+2)$ when $x = -4$?
- 3.2) Could $(x+1)(x+2)$ ever be a negative number? Explain.
- 3.3) Solve this equation for x . Show your steps. $5x - 6 = 4x + 3$
- 3.4) Describe two ways you could check your answer to the previous question.

Sample Assessment Tasks

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?
- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

Quantifying Understanding

- Task-specific rubrics are needed for evaluating the degree of understanding: (0 = no, 1 = low, 2 = moderate, 3 = high)
 - Tasks 3.1 & 3.2
 - Tasks 3.3 & 3.4
 - Task 3.15 and 3.16
- What do you notice about the student responses?

Rubric for Item 3.15

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?
 - 0: Blank or no honest effort (e.g. "I don't know (IDK.)")
 - 1: Two or more mistakes present OR an incomplete, illogical, confusing, or way off track response.
 - 2: Only one mistake with an attempted answer.
 - 3: Correct answer is provided.

Rubric for Item 3.16

- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?
 - 0: Blank or no honest effort (e.g. "I don't know (IDK.)")
 - 1: Response is vague, illogical, confusing, or way off track.
 - 2: States "they will intersect" OR "it tells you whether they intersect or are parallel."
 - 3: States "they will intersect at $x = -2$ " (or previous problem's x value.)

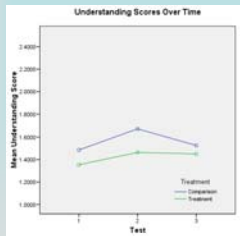
Sample Responses – Item 3.16

- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

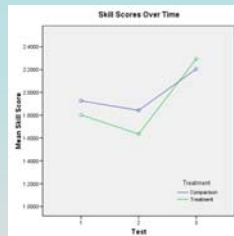
The lines would touch at $(-2, 5)$ if you were to graph them. Scored: 3

They will have the same x values but probably different slopes Scored: 2

Results – Understanding & Skill



Gains in understanding from test one to test three...
Test1 → Test 3, $p=.413$ (n.s.)



Gains in skill from test one to test three...
Test1 → Test3, $p = .003$

A Closer Look – Number Sense

- 3.1) What is the value of the expression $(x+1)(x+2)$ when $x = -4$?
- 3.2) Could $(x+1)(x+2)$ ever be a negative number? Explain.

		q3.2 (understanding)					
		Score	0	1	2	3	Total
q3.1 (skill)	0	9	1	0	0	0	10
	1	3	28	3	1	0	35
	2	1	10	3	0	0	14
	3	2	150	40	6	0	198
Total		15	189	46	7	0	257

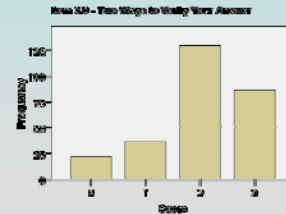
A Closer Look – Verify Answer

- 3.3) Solve this equation for x . Show your steps.
 $5x - 6 = 4x + 3$
- 3.4) Describe two ways you could check your answer to the previous question.

		q3.4 (understanding)					
		Score	0	1	2	3	Total
q3.3 (skill)	0	4	1	0	0	0	5
	1	7	11	16	6	0	40
	2	1	2	5	0	0	8
	3	6	14	162	22	0	204
Total		18	28	183	28	0	257

A Closer Look – Best Method?

- 3.8) Comparing Tony's and Alisha's methods: Is one way better? Will they both always work?
– For full credit (3), needed to affirm both always work and discuss simplicity / efficiency.



A Closer Look – Utility / Meaning

- 3.15) If $3x + 4y = 14$, and $y = -(5/2)x$, what is the value of x ?
- 3.16) What does the answer to the previous problem tell you about the graphs of the two equations?

		q3.16 (understanding)					
		Score	0	1	2	3	Total
q3.15 (skill)	0	40	5	0	0	0	45
	1	35	73	11	1	0	120
	2	5	36	3	0	0	44
	3	8	32	5	3	0	48
Total		88	146	19	4	0	257

Now, Back to You

- (Re)consider the task you wrote on your index card...
- Write a revision or a follow-up that you could use to deepen or enrich the nature of the task.
- Th(ink)-Pair-Share

Conclusion & Discussion

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- Students do not automatically develop (or communicate?) deep knowledge.
- These “habits of mind” should be modeled during instruction and valued during practice & assessment.
 - Focus is needed on habits such as communication, multiple representations, flexibility and alternate methods.
- What “habits of mind” are most important for success in your class? In your work? In life?