Incorporating Writing in an Integrated Calculus, Linear Algebra and Differential Equations Sequence

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Abstract

In many universities across the country writing is being stressed throughout the curriculum. Faculty see the need to expose students to more writing and to teach the students the correct writing styles for their disciplines. Since 1991 the University of Wisconsin-La Crosse has required students to take a writing intensive course in their major. The Mathematics Department first addressed this requirement by offering various courses as writing intensive; instructors would decide when they wished to teach a course with a writing emphasis. There were various drawbacks to this system, and in 1998 the Department changed its format to a writing intensive sequence. Writing is incorporated throughout a four-semester sequence integrating calculus, linear algebra and differential equations.

In this paper the specific courses in this sequence are described along with how the writing has been implemented in each course. Ideas are also given for how to efficiently handle the additional paper load so students receive the necessary feedback while keeping the grading time reasonable.
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1 Writing Program

Writing-across-the-curriculum has become widely popular in recent years. Universities are recognizing the importance of writing skills for their students. After graduation, students will be expected to communicate both verbally and in writing with a variety of people including perhaps their bosses, colleagues and the general public. For this reason it is important for students to learn the appropriate writing styles and techniques both within and outside their discipline.

[1]**Key Words:** Writing, Calculus, Linear Algebra, Differential Equations, Curriculum

In mathematics students need to learn to incorporate mathematical language and notation with English text. By formally writing out their work, the students also gain a better understanding of the concepts and learn the difference between a final answer and a solution.

To better educate students in writing, the University of Wisconsin-La Crosse instituted a writing requirement as part of its General Education Program; each student must complete two courses designated as writing intensive courses, one of which must be in the student's major. Faculty attend a workshop in which they are introduced to the specific requirements of a writing intensive course, as defined by the General Education Program, and then complete an application process to become certified to teach writing intensive courses. Once certified, instructors may chose to offer any of their courses as writing intensive courses.

In the Mathematics Department's first attempt to meet the writing requirement, individual instructors decided when they wished to have one of their courses designated as a writing intensive course for a particular semester. There were several problems with this approach. It was difficult to incorporate enough writing in a single course to truly meet the definition of a writing intensive course while still covering all the course content. Exposing students to writing in mathematics for only a single semester did not give students enough time to learn what mathematical writing is all about. Without a consistent schedule of writing intensive courses in mathematics, our students sometimes had difficulties in fulfilling this General Education requirement.

In 1998, in an attempt to better serve our students, the Mathematics Department moved from single writing intensive courses to a writing intensive sequence. Rather than trying to incorporate the writing in a single course, we spread the writing throughout a four-semester entry-level sequence. The sequence was introduced a year earlier to integrate calculus, linear algebra and differential equations. We believe incorporating writing through several courses is much better than the single course approach. When writing is added to a particular course, students tend to see it as just a few assignments in that course. By requiring students to write in several courses, they begin to see writing as an integral part of mathematics just as the sequence shows the students how topics in mathematics are related. Students also get a chance to write on various topics and receive feedback from more than one instructor as they progress through the sequence. In addition, by spreading the writing throughout a sequence, the time devoted to writing in any one course is more manageable. All of these features make for a much better

introduction to mathematical writing.

Since we vary instructors for courses in our Department, most of the twenty-three faculty members teach courses in this sequence at some time. To help unify the approach we have to writing, a committee wrote a formal Writing-In-The-Major Proposal [10] that was approved by the Department and the University of Wisconsin-La Crosse's General Education Committee. The proposal included the following points: (1) forms of formal writing students will be expected to learn, such as technical reports, expository papers, and elementary proofs which will be discussed in the following section; (2) an outline of how our program is structured to advance writing skills as students progress through the sequence, as discussed in Section 2; (3) shared criteria for faculty on how to evaluate student writing along with helpful points on managing the additional grading, as discussed in Section 3; (4) a process for assessing the quality of the program, as outlined in Section 4.

2 Writing In the Sequence

The sequence in which we have incorporated writing is a four-semester sequence integrating calculus, linear algebra and some differential equations. One goal for this sequence is to help students see the connections between these areas rather than view them as discrete topics. The topics are mixed through the semesters as described later. This is the entry-level sequence in our mathematics major. Some of the courses are also required for other majors such as chemistry (first three courses) and physics (first three courses for some areas and all four courses for other areas). By requiring this sequence as a prerequisite to most of our upper level mathematics courses, we insure a consistent background in both mathematics and writing for all of our junior and senior level students.

Writing assignments for these courses can come from project books or from extending ideas from standard texts. Specific examples and references will be given for each course in the sequence. Although each instructor has some freedom concerning the number and type of assignments they give, typically three to five writing assignments would be given each semester in addition to the usual textbook homework assignments. The writing assignments typically account for ten to fifteen percent of the course grade. These assignments are evaluated in part based on standard grading criteria discussed in Section 3. Individual instructors retain the flexibility to emphasize these criteria as they see fit and may also add additional criteria

appropriate for specific assignments.

Different instructors handle the details of the assignments differently. In some cases students work in teams on the writing assignments. This is one way to cut down on the amount of grading required and still have the students doing some writing. It can also be helpful to the students to discuss the problem with someone. Having two people to proof-read the paper is also a plus. In other cases, usually only on the longer assignments, students first hand in a rough draft on which they receive feedback. A final draft is then due approximately one week after the rough draft is returned. This approach, although more time consuming for the instructor, gives the student the opportunity to apply the feedback they receive. Another version of this approach is to have students first hand in just the mathematical steps in the solution of the problem. This can then be checked for correctness and completeness before the student begins writing up their explanations and conclusions. This approach helps the students to pace their work instead of pushing it all off until shortly before the assignment is due.

Many assignments involve some use of technology. This allows the students to examine more involved problems with more realistic conditions. The technology helps them with messier computations and with more detailed graphs.

Since the writing is taught in a sequence, the expectations increase as the students progress through the courses. The types of assignments are designed to reflect the students' level of development. The specific types of assignments for each course will be explained below.

2.1 Course One

The first course in our sequence is a standard five-credit first-semester calculus course. The topics covered include limits, differentiation and basic integration. In this course we begin the students' introduction to writing in mathematics using homework problems that require students to explain, interpret or analyze their solution to a problem rather than just writing out the mathematical steps. The problems are familiar, but they are asked to describe the solution process in English as well as using mathematical notation. These assignments are typically no more than a page or two in length. They are intended to increase the students' comfort with a particular mathematical idea and get them to begin writing about mathematics for a specific audience. These types of problems can be found in most textbooks, but may also come from project books such as [2] and [6]. In many

cases, with a simple change in instructions, a standard exercise can become a writing assignment.

One example of a standard type of problem that can be modified to include writing is a problem where the students are asked to match the graphs of functions with the graphs of their derivatives. By asking the students to explain their choices, this problem becomes a writing exercise. It also forces the students to better understand their answers because they need to explain their reasoning. They might be asked to write as if explaining to someone who missed class.

Another example of an assignment that has worked well for this course is based on an article in the American Mathematical Monthly [1]. The article discusses the calculus behind landing an airplane at the Lake Tahoe airport. You can use the ideas in this article to have students model the landing approach of an airplane. This problem can be given relatively early in the semester and reinforces important concepts including tangent lines, the chain rule and higher-order derivatives. A version of this problem also appears as an Applied Project in the latest edition of J. Stewart's calculus text [7, p. 199].

Other good types of problems for these short writing assignments are the standard optimization and related rates problems. For instance, one such problem is well laid out in [7, pp. 287-288]. In this problem students are asked to investigate the most economical shape for a can. In a typical homework problem, students would be told to ignore any waste material in the manufacturing process. In this project, students are asked to include the waste material in their considerations. This makes for a more challenging and more interesting problem for students. It is also a more realistic problem, a fact not lost on them. Many students find optimization and related rates problems particularly difficult. Having to describe the problems and carefully explain their solutions forces them to think more carefully about them. These types of writing assignments also help the students to begin to see the difference between final answers and complete solutions. The need for complete solutions might be made more relevant by asking them to put their results in the form of a short technical report to a boss who is not a mathematician.

2.2 Course Two

The second course is a four-credit course that covers exponential and logarithmic functions, integration techniques and applications, first order dif-

ferential equations and an introduction to matrices and linear systems. In this course the writing assignments typically involve the solution of a more realistic (and hence more involved) application of the mathematical content. Students are expected to write an appropriate introduction to the problem and give a thorough explanation of their solution. This involves incorporating mathematical notation and any needed graphs into the English text. Finally the students are required to write a conclusion in which they interpret their solution and relate it back to the original problem. Typically these applications would involve only one major mathematical concept and be a few pages in length.

One interesting assignment for this course is to have the students use indefinite integrals to explain why a rocket can leave the earth's gravitational pull but can not escape a black hole. Mathematically this is explained with improper integrals. Since the force of gravity is proportional to the reciprocal of the square of the distance between the bodies, the energy needed to leave the earth's gravitational pull requires integrating $1/x^2$ from the radius of the earth to infinity, a finite value. To escape a black hole, the integral must start at zero and thus is an infinite value. For greater detail on this idea see [9, pp. 387-397]. This assignment would be a good one to have students write for an audience of other calculus students who are not working on this project so that complete explanation of the problem is required.

One possible sequence of writing assignments that helps integrate ideas from calculus and differential equations is to have students use various models to represent the population of the United States at various times in history. These writing assignments were created by simply extending topics presented in many calculus and differential equations texts. Students are given census data for various times in history. As an example, the census data presented in [4, p. 125] can be used. From this data they can be asked to estimate derivatives representing relative growth rates, use exponential and logistic functions to model the population and give historical reasons for when the models fail. This work not only illustrates how the mathematics is used, but also illustrates that there may not always be one perfect model or only one possible solution. In this assignment students could be asked to write for a more general audience.

2.3 Course Three

The third course is a four-credit course covering topics including R^n as a vector space, eigenvalues and eigenvectors, systems of two first order differential

equations and sequences and series. With the linear algebra included in this course, this is an excellent place to introduce the writing of proofs. The goal for writing in this course is to introduce students to the basic structure of proofs and have them begin to write simple proofs on their own.

The types of proofs that students would be asked to write at this level would be rather elementary. Many proofs in linear algebra require primarily checking whether certain properties or definitions are satisfied. For example, students might be asked to prove that a collection of vectors is a basis for R^3 or that a certain set of vectors is a subspace of R^n. Another example might be to verify certain properties of the null set.

Students would not be expected to master proof writing in this course. They will continue to learn proof writing techniques in the advanced courses beyond this initial sequence. This course is considered the students' first introduction to this particular type of mathematical writing.

2.4 Course Four

The final course in the sequence is a four-credit course that covers topics from multivariable calculus. The writing assignments for this course are similar to those given in course two as far as their format. They would be on more advanced applications of mathematics, but would again require an appropriate introduction and conclusion as well as a thorough explanation of the solution. These assignments typically will involve several mathematical concepts, and require students to integrate ideas that they have studied in the previous courses. The final project is typically several pages in length and is often required to be in a form which resembles the structure of an article in an undergraduate mathematics journal. Students are asked to do fewer writing assignments as the length of assigments increase.

As examples, students may be asked to extend the ideas they learned in the first course to help explain how extrema problems in two variables work. Similar work could be done with Lagrange multipliers. Such assignments would use optimization ideas from the first course with the new work in higher dimensions. The assignments would also apply some of the logical proof structure they learned in the third course to different areas of mathematics.

Some very nice ideas for writing assignments for this material can be found in the Applied and Discovery Projects given in [7]. One nice project, [7, p. 983], has students locate a trash dumpster and carefully study the details of its construction and determine its volume. They are then given

costs for the materials used for various parts along with welding costs. The students are asked to determine the dimensions of a dumpster of the same volume and general shape which minimizes the cost. They are also asked to describe and justify any simplifications that they used and how these simplifications affected the results. The problem goes on to ask the students if they would want to alter the general shape of the dumpster. This project uses partial derivatives in a more involved extrema problem than those they would have worked on in the first calculus course.

As another possible writing assignment, students could be asked to prove $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$. This sum was proved by Euler and can be evaluated using double integrals. In working through this proof the students will also need to revisit trigonometric functions and infinite series. This work incorporates new ideas and also skills learned in earlier courses in the sequence. It also requires the students to use some of their skills from proof writing. This assignment is based on a more involved problem given in [7, p. 1072].

3 Evaluating Writing Assignments

Although each assignment may have different specific goals and will be evaluated in part based on these, some general criteria are used for the evaluation of all writing assignments. Instructors are free to add to, but not subtract from the list of shared criteria. The following list gives the general shared criteria and the descriptions supplied give the highest standard for that criterion [10]: (1) The student should identify the problem. The paper should contain a specific statement of the problem to be addressed by mathematical means. If there are sub-problems, the paper describes how solving each sub-problem addresses the main problem. (2) Mathematical correctness and completeness is expected. The paper includes a complete solution to the mathematics problem without error. It contains a description of each step in the solution process so as to produce a logical flow from the statement of the problem to the stated conclusion. (3) The student should interpret the solution. (This criterion would not be included when evaluating proofs.) The paper clearly describes the usefulness of the solution of the mathematics problem as it relates to the original question. It explains the relation of all sub-problem solutions to the entire problem. (4) The clarity in communication of mathematics is checked. The paper integrates both precise and appropriate language, and precise and appropriate notation with clear English text. (5) The level of mathematics presented in the paper is

Figure 1: Sample Evaluation Sheet

appropriate for the given audience. The explanations appropriately utilize (without justification) only a certain body of knowledge that the audience is assumed to possess. (6) The paper should be well organized and easy to follow. The outline and style of the paper is consistent with the given style and includes all components specified. The transitions from topic to topic facilitate understanding of major ideas. (7)Basic writing skills including spelling, grammar, and punctuation should be flawless. Graphs, tables, and other referenced information are inserted in, or appended to, the paper appropriately.

Now that we have given ideas for a writing intensive sequence and the shared criteria on which they will be evaluated, you may be wondering how to handle the additional grading. One way to manage the work is to design evaluation sheets (see a sample in Figure 1). This idea combines techniques of instructors at our institution and ideas given by B. Walvoord [8].

For each assignment, an evaluation sheet is written. This sheet would contain a chart listing various items that will be checked in the paper and the number of points each item is worth. This listing includes both mathematical steps and writing quality. Mathematical items could include specific questions asked in the assignment and appropriateness of details given to explain solutions. The writing points could include a proper introduction and conclusion, writing to the appropriate audience and the use of correct

grammar and spelling. Sometimes an instructor may wish to give this sheet to students while they are working on their assignments to help them focus their efforts on the correct areas. Other times it may be more appropriate to only give this to students when their graded papers are being returned.

Below such a chart the instructor lists common comments that they expect they will use when grading the assignments. When grading each paper, the instructor simply circles the comments which pertain to that paper. This part has been particularly useful in saving time by not having to write the same comment over and over. Below these general comments, other specific comments can be added by hand.

In practice, the evaluation sheet appears to have several benefits. The students seemed to take the typed comments more seriously than hand written ones and later papers show greater improvement. In addition, the students seemed pleased to know exactly how they were graded. This method also saves the instructor a great deal of time in grading by making the process more systematic.

4 Assessment

Since our new approach to writing is only two years old, we have not yet completed a formal assessment of the writing program or the integrated mathematics sequence. The integrated aspect of the core sequence is currently undergoing a review process. Some initial assessment has also been done on the writing aspect of the sequence.

Each semester, all instructors teaching in the core sequence (and other interested faculty) meet to discuss how the courses are going. We share writing assignments, ideas on grading, and concerns. From these meetings, we have at least anecdotal evidence that our Writing-In-the-Major program is accomplishing some of its goals. Instructors in the later courses of the sequence indicate fewer problems with details in student papers such as identifying the problem, introductions and conclusions, and writing to the appropriate audience.

As the start of our formal assessment, each year members of our Department review samples of student writing from the courses in the sequence. At least three Department members review each sample, using the shared criteria stated in Section 3. The results of these reviews are used to determine whether or not the students are meeting the Department's expectation that they should be able to look at a new problem, understand the mathematical

aspects of the problem, analyze and solve the problem, and explain their work in a clear manner.

A summary of the results of this review is distributed to the Department. If it is determined that students are not meeting the Department's expectations, the results of the review are used to identify actions the Department might take to address the apparent weaknesses in the students' work. So far these reviews of student work have indicated that the program seems to be working. In the future, we will be looking at the effect of the writing intensive sequence on later courses in the major. Certain upper level courses consistently have students working on projects and writing reports. Instructors of these courses will be asked whether they see an improvement in the students' writing once the students have gone through the writing intensive sequence. At this point the writing program has not been in effect long enough for the students to reach the upper level courses.

5 Conclusion

This paper presents some of the ideas we have used for our writing program. The main point to our program is to give the students several semesters to begin to develop their writing skills. Requiring appropriate writing assignments in many upper level courses then sharpens these skills

We have given ideas of types of writing assignments that could be used and ideas to help with the additional grading. Included in the bibliography are a few references which we have found helpful. As suggested in the paper, we created many of the writing assignments by simply elaborating on ideas given in texts.

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