Exam 2 Review

- 4.3 Properties of linear transformations
  - What is a linear transformation? (Definition)
  - Be able to demonstrate a transformation is/isn't linear.
  - Matrix of a linear transformation.
  - Given a description (rotation, reflect, etc.) be able to find matrix $[T(\vec{e}_1)...T(\vec{e}_n)]$
  - New pieces of the snowball theorem: range of T_A is $\mathbb{R}^n$ and T_A is one-to-one.
  - Be able to show a transformation or function is one-to-one or not.
  - What does one-to-one mean? (Definition)
  - Should know the snowball theorem up to this point.
- 5.1 Vector spaces
  - Don't need to memorize the 10 axioms
  - Given a list of the 10 axioms you should be able to verify them or show which ones are broken
- 5.2 Subspaces
  - What is a subspace? (Definition)
  - How to determine if a vector is in a subspace?
  - How to verify that a subset is a subspace? (Theorem)
  - How to show a subset is not a subspace?
  - Special subspaces including null space (solution space of $A\vec{x} = \vec{0}$) and span of a set of vectors
  - Does a given set of vectors span a subspace?
  - Subspaces of M_{mn}
- 5.3 Linear Independence
  - What is a linearly independent set? (Definition)
  - What is a linearly dependent set? (Definition)
  - How do you determine if a set of vectors is linearly independent?
  - Theorems 5.3.1, 5.3.2, and 5.3.3 are good results to be a little familiar with
  - A set of vectors that spans a subspace need not be linearly independent
  - A set of linearly independent vectors need not span a subspace
- 5.4 Basis and Dimension
  - What is a basis of a vector space V? (Definition)
  - What is the dimension of a vector space V? (Definition)
  - How are linearly independent set, spanning set, and dimension related? (Theorem)
  - How do you determine if a set of vectors is a basis if you don't know the dimension of the vector space?
  - How do you determine if a set of vectors is a basis if you do know the dimension of the vector space?
  - What are standard basis vectors?
  - Is a basis unique?
  - Quick checks for linear independence and spanning that use dimension.
  - $\dim(\mathbb{R}^n), \dim(M_{mn}), \dim(P_n)$

- 5.5 Row, Column, and Null spaces
  - what are these spaces? (Definitions)
  - If A is $m \times n$ then the subspaces are in $\mathbf{R}^n$ or $\mathbf{R}^m$, which?
  - understand Theorem 5.5.1 page 267
  - general form of solutions to $A\vec{x} = \vec{b}$
  - what is a particular solution?
  - how to get general form?
  - how to get null space basis from general form of solution?
  - row equivalent matrices have the same row spaces (why?)
  - finding a basis for the row space in two ways
    - put into row-reduce echelon form, use nonzero row vectors
    - basis in terms of orignial rows
  - finding any basis for a set of vectors (put vectors in rows of matrix and row reduce - look for leading ones)
  - row equivalent matrices do not have the same column spaces (why not? give example)
  - Column spaces of row equivalent matrices are connected because if $A \sim B$ then $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ have exactly the same solutions.
  - To find a basis for the column space that is a subset of the original columns, row reduce and look for leading ones
  - To reduce a set of vectors to a basis put those vectors into columns of matrix, row reduce, use vectors corresponding to columns with leading ones (also can find dependence relationships)
- 5.6 Rank and Nullity
  - row and column space of A have the same dimension!
  - fundamental matrix subspaces (there are 4, what are they?)
  - Definitions - rank and nullity
  - $\text{rank}(A) = \text{rank}(A^T)$
  - Connections to leading and free variables in $A\vec{x} = \vec{0}$
  - $\text{rank}(A) + \text{nullity}(A) = n$