Math 309 – Dr. Peirce   Exam 3 [67 points]   Fall 2009

Name Solutions

READ FIRST. No calculators, books, notes or other written material allowed. Read each question very carefully. Show your work neatly for partial credit. To receive full credit your work should be correct, organized, include all supporting calculations, and clearly indicate your final answer. Correct answers with too few supporting calculations will receive little or no credit.

1. [9 points] Consider the subspace W of $\mathbf{R}^3$ spanned by $\{\vec{u}, \vec{v}\}$ given by

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

a. Show that $\vec{u}$ is orthogonal to $\vec{v}$.

$$\vec{u} \cdot \vec{v} = 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 = 0 \quad \checkmark$$

+1

b. Express $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ as the sum of two vectors $\vec{w}_1 + \vec{w}_2$ where $\vec{w}_1 \in W$ and $\vec{w}_2 \in W^{\perp}$

+2 +2

$$\vec{w}_1 = \frac{\langle \vec{w}, \vec{u} \rangle}{\|\vec{u}\|^2}\vec{u} + \frac{\langle \vec{w}, \vec{v} \rangle}{\|\vec{v}\|^2}\vec{v} = 0 + \frac{1}{(\sqrt{2})^2}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix} \qquad +1$$

$$\vec{w}_2 = \vec{w} - \vec{w}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/2 \\ -1/2 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2 \\ -1/2 \end{bmatrix}$$

+2 +1

2. [7 points] Let V be an inner product space. Show that if $\vec{u}$ and $\vec{v}$ are othonormal vectors in V, then $||u - v|| = \sqrt{2}$.

+2

$$\|\vec{u} - \vec{v}\|^2 = \langle \vec{u} - \vec{v}, \vec{u} - \vec{v} \rangle$$

$$= \|\vec{u}\|^2 - 2\langle \vec{u}, \vec{v} \rangle + \|\vec{v}\|^2 \qquad +2$$

$$= 1 - 2(0) + 1 = 2 \quad \longrightarrow \quad \|\vec{u} - \vec{v}\| = \sqrt{2}.$$

+2 +1