Name Solutions **Math 309 Quiz 9**

- The matrix A on the left row reduces to the matrix on the right (believe me):

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 & 2 & 3 \\ 2 & -2 & 2 & 4 & 6 \\ 1 & -1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

a. Find a basis for the orthogonal compliment of null(A)

$(\text{null}(A))^\perp = \text{row } A \longrightarrow$ basis of row space is $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

b. Find a basis for the orthogonal compliment of null(A^T)

$(\text{null } A^T)^\perp = \text{col } A \longrightarrow$ basis of column space is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

- Find a basis for the orthogonal compliment of the subspace of $\mathbb{R}^3$ spanned by the given vectors.

$$S = \left\{ \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -9 \\ 4 \end{bmatrix} \right\}$$

If $\vec{v} \in \text{span } S$ then $\vec{v} = k_1 \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} + k_2 \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix} + k_3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + k_4 \begin{bmatrix} 2 \\ -9 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -1 & 0 & 2 \\ -5 & 5 & 1 & -9 \\ 2 & -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} \longrightarrow \vec{v} \in \text{col }(A)$$

Since $(\text{col }(A))^\perp = \text{null}(A^T)$ we need to find basis for null A^T:

$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & 0 \\ -1 & 5 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & -9 & 4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -5 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$x_2 = 0$

$x_1 + 2x_3 = 0 \qquad x_1 = -2x_3$

$$\vec{x} = \begin{bmatrix} -2t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} t$$

basis is $\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$