MTH 309 Fall 2009

Project 2: Due Monday Dec 7th

Proofs for Points

Complete 5 of the following proofs for 5 points each. Each proof should be on a separate page and should include the statement you are trying to prove.

1. Suppose V is an inner product space with dimension n. Prove that any set of n nonzero orthogonal vectors form a basis for V.

2. Let $B = \{\vec{u}_1, \vec{u}_2, \ldots \vec{u}_p\}$ be an orthonormal basis for a subspace W. Let v be any vector in W, where $v = k_1\vec{u}_1 + k_2\vec{u}_2 + \ldots + k_p\vec{u}_p$. Prove that

$$||\vec{v}||^2 = k_1^2 + k_2^2 + \ldots + k_p^2.$$

3. Let A be an $(n \times n)$ matrix, and let λ be an eigenvalue of A. Prove that if α is any scalar, then $\lambda + \alpha$ is an eigenvalue of $A + \alpha I$.

4. Let $\vec{u}$ and $\vec{v}$ be vectors in an inner product space. Prove that $||\vec{u}|| = ||\vec{v}||$ if and only if $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal.

5. Prove that the map $T : P_n \to P_n$ defined by $T[p(t)] = \dfrac{dp}{dt}$ a linear transformation and determine if T is one-to-one.

6. Let $T_A : \mathbb{R}^n \to \mathbb{R}^m$. Prove that the range of T_A is a subspace of $\mathbb{R}^m$.

7. Show that the set of all polynomials in P_n that have a horizontal tangent at $x = 0$ is a subspace of P_n. Find a basis for this subspace.