Determine the following limits, if they exist.

1. $\lim_{x\to 1}\frac{2x^2+x-3}{x^2-1}$ $= \lim_{x\to 1} \frac{(2x+3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \lim_{x\to 1} \frac{2x+3}{x+1} = \frac{5}{2}$

2. $\lim_{h\to 0}\frac{(2+h)^3-8}{h}$ $= \lim_{h\to 0} \frac{(2^3 + 2^2\cdot h \cdot 3 + 2\cdot h^2 \cdot 3 + h^3) - 8}{h} = \lim_{h\to 0} \frac{12h + 6h^2 + h^3}{h}$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= \lim_{h\to 0} 12 + 6h + h^2 = 12$

3. $\lim_{x\to -1}\frac{2-x}{(x+1)^2}$

↳ looks like $\frac{3}{0}$

Note $(x+1)^2 \geq 0$

Since $2-x > 0$ near $x=-1$ and $(x+1)^2 \geq 0$,

$\lim_{x\to -1} \frac{2-x}{(x+1)^2} = \infty$

4. $\lim_{x\to 5^-}\frac{x-7}{x-5}$

If I plug in $x=5$, I get $\frac{-2}{0}$

As $x \to 5^-$, $x-5 \to 0^-$ → So, $\lim_{x\to 5^-} \frac{x-7}{x-5} = +\infty$

5. $\lim_{t\to -3}\frac{t^2-9}{2t^2+7t+3}$ $= \lim_{t\to -3} \frac{(t-3)\cancel{(t+3)}}{(2t+1)\cancel{(t+3)}} = \lim_{t\to -3} \frac{t-3}{2t+1} = \frac{-6}{-5} = \frac{6}{5}$

If I plug in $t=-3$, I get $\frac{0}{0}$.

6. $\lim_{t\to 9}\frac{9-t}{3-\sqrt{t}}$ $= \lim_{t\to 9} \frac{9-t}{3-\sqrt{t}}\left(\frac{3+\sqrt{t}}{3+\sqrt{t}}\right) = \lim_{t\to 9} \frac{\cancel{(9-t)}(3+\sqrt{t})}{\cancel{(9-t)}} = \lim_{t\to 9} 3+\sqrt{t}$

$= 6$

7. $\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x}$ $= \lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}\left(\frac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}\right)$

$= \lim_{x\to 0} \frac{(\cancel{3}+\cancel{x}-\cancel{3})}{\cancel{x}(\sqrt{3+x}+\sqrt{3})} = \lim_{x\to 0} \frac{1}{\sqrt{3+x}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$

8. $$\lim_{x\to-4}\frac{\frac{1x}{4x}+\frac{1}{x}\cdot\frac{4}{4}}{x+4} = \lim_{x\to-4}\frac{\frac{x+4}{4x}}{x+4} = \lim_{x\to-4}\left(\frac{\cancel{x+4}}{4x}\right)\cdot\left(\frac{1}{\cancel{x+4}}\right)$$
$$= \lim_{x\to-4}\frac{1}{4x} = -\frac{1}{16}$$

9. $$\lim_{x\to-\infty}\frac{4x^3-2x+3}{x^3-3x^4}\cdot\left(\frac{\frac{1}{x^4}}{\frac{1}{x^4}}\right) = \lim_{x\to-\infty}\frac{\frac{4}{x}-\frac{2}{x^3}+\frac{3}{x^4}}{\frac{1}{x}-3} = \boxed{0}$$

10. $$\lim_{x\to-\infty}\frac{\sqrt{4x^2+1}}{3-2x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty}\frac{\sqrt{4+\frac{1}{x^2}}}{\frac{3}{x}-2} = \frac{\sqrt{4}}{-2} = -1$$

11. $$\lim_{x\to\infty}\frac{\sqrt{9x^2-16}-\sqrt{4}}{x-2}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty}\frac{\sqrt{9-\frac{16}{x^2}}-\sqrt{\frac{4}{x^2}}}{1-\frac{2}{x}} = 3$$

12. $$\lim_{x\to\frac{5\pi}{6}}\frac{\cos x}{\sin 2x} = \frac{\cos\left(\frac{5\pi}{6}\right)}{\sin\left(\frac{5\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2}} = 1$$

13. $$\lim_{\theta\to0}\frac{\tan 2\theta}{3\theta} = \lim_{\theta\to0}\frac{2}{2}\left(\frac{\sin 2\theta}{\cos 2\theta}\right)\frac{1}{3\theta} = \lim_{\theta\to0}\underbrace{\left(\frac{\sin 2\theta}{2\theta}\right)}_{\to 1}\underbrace{\frac{2}{3\cos 2\theta}}_{\to 1} = \frac{2}{3}$$

14. $$\lim_{x\to0}\frac{2}{x\cot x} = \lim_{x\to0}\frac{2\tan x}{x} = \lim_{x\to0}2\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{x}$$
$$= \lim_{x\to0}\underbrace{\frac{2}{\cos x}}_{\to 2}\cdot\underbrace{\frac{\sin x}{x}}_{\to 1} = 2$$

15. $\lim_{x\to 0}\frac{\sin 3x}{\sin 2x\cos x}$ $=\lim_{x\to 0}\left(\frac{\cancel{3}\sin 3x}{1}\cdot\frac{1}{\sin 2x}\cdot\frac{1}{\cos x}\right)\left(\frac{3x}{3x}\right)\left(\frac{2}{2}\right)$

$\lim_{x\to 0}\frac{\sin x}{x}=1$

$=\lim_{x\to 0}\underbrace{\frac{\sin 3x}{3x}}_{\to 1}\cdot\underbrace{\frac{2x}{\sin 2x}}_{\to 1}\cdot\frac{3}{2\cos x}=\frac{3}{2}$

16. $\lim_{t\to 0}\frac{|t|+t}{t}$

$=\lim_{t\to 0}\left(\frac{|t|}{t}+1\right)$

$=\left(\lim_{t\to 0}\frac{|t|}{t}\right)+1$

$\to$ DNE

$|t|=\begin{cases} t & \text{if } t\geq 0\\ -t & \text{if } t<0\end{cases}$

$\frac{|t|}{t}=\begin{cases} 1 & \text{if } t>0\\ -1 & \text{if } t<0\end{cases}$

$\lim_{t\to 0^-}\frac{|t|}{t}=-1 \qquad \lim_{t\to 0^+}\frac{|t|}{t}=+1$