MTH 207 – Calculus I
Quiz 3
September 26, 2008

Name: ______________________
Section: ________

Answer each question to the best of your abilities. Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Good luck!

1. (*5 points*) Show that the equation

$$12x^6 - 4x^5 + 9x^4 + 3x^3 - 7x^2 - x - 1 = 0$$

has at least one solution between 0 and 1 (cite any theorems you invoke).

Let $f(x) = 12x^6 - 4x^5 + 9x^4 + 3x^3 - 7x^2 - x - 1$. f is continuous.

$f(0) = -1$

$f(1) = 12 - 4 + 9 + 3 - 7 - 1 - 1 = 11$

Note $f(0) < 0$, and $f(1) > 0$. (i.e. $f(0) < 0 < f(1)$.

By IVT, there exist $c \in (0,1)$ such that $f(c) = 0$

2. (*5 points*) Calculate

$$\lim_{x\to\infty} \frac{3x^2 + 2x + 1}{2 + 6x + 7x^2}.$$

Show all work!

$$\lim_{x\to\infty} \left(\frac{3x^2+2x+1}{2+6x+7x^2}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) = \lim_{x\to\infty}\left(\frac{3+\frac{2}{x}+\frac{1}{x^2}}{\frac{2}{x^2}+\frac{6}{x}+7}\right) = \frac{3}{7}$$

3. (*5 points*) Use the definition to calculate the derivative of $f(x) = x^2 + 4x + 6$ at the point $x = 2$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \lim_{h\to 0} \frac{[(2+h)^2 + 4(2+h) + 6] - [4+8+6]}{h}$$

$$= \lim_{h\to 0} \frac{4 + 4h + h^2 + 8 + 4h + 6 - 18}{h} = \lim_{h\to 0} \frac{4h + h^2 + 4h}{h}$$

$$= \lim_{h\to 0} 8 + h = \boxed{8}$$