

Answer each question to the best of your abilities. Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Good luck!

1. (5 points) Let $f(x) = 4x^5 - 3x^3 + 2x^2 + 5e^x - 1$. Find $f'(x)$.

$$\begin{aligned} f'(x) &= 5 \cdot 4 \cdot x^4 - 3 \cdot 3x^2 + 2 \cdot 2x + 5e^x \\ &= \boxed{20x^4 - 9x^2 + 4x + 5e^x} \end{aligned}$$

2. (5 points) Let $g(x) = \left(\frac{1}{x} - 2x\right)(\sqrt{x} - 3x^2)$. Use the product rule to find $g'(x)$.

$$\begin{aligned} g(x) &= (x^{-1} - 2x)(x^{1/2} - 3x^2) \\ g'(x) &= (x^{-1} - 2x) \frac{d}{dx}(x^{1/2} - 3x^2) + (x^{1/2} - 3x^2) \frac{d}{dx}(x^{-1} - 2x) \\ &= (x^{-1} - 2x) \left(\frac{1}{2}x^{-1/2} - 6x\right) + (x^{1/2} - 3x^2)(-x^{-2} - 2) \\ &= \frac{1}{2}x^{-3/2} - 6x^{1/2} + 2x^2 + -x^{-3/2} - 2x^{1/2} + 3 + 6x^2 \\ &= -\frac{1}{2}x^{-3/2} - 3 - 3x^{1/2} + 18x^2 \\ &= \boxed{18x^2 - 3\sqrt{x} - 3 - \frac{1}{2\sqrt{x^3}}} \end{aligned}$$

3. (5 points) Let $h(x) = \frac{x+2}{x^3+x-2}$. Use the quotient rule to find $h'(x)$.

$$\begin{aligned} h'(x) &= \frac{(x^3+x-2) \frac{d}{dx}(x+2) - (x+2) \frac{d}{dx}(x^3+x-2)}{(x^3+x-2)^2} \\ &= \frac{(x^3+x-2)(1) - (x+2)(3x^2+1)}{(x^3+x-2)^2} \\ &= \frac{x^3+x-2 - [3x^3+6x^2+x+2]}{(x^3+x-2)^2} \\ &= \frac{-2x^3-6x^2-4}{(x^3+x-2)^2} = \boxed{\frac{-2(x^3+3x^2+2)}{(x^3+x-2)^2}} \end{aligned}$$