

Answer each question to the best of your ability. Show all work clearly and in order, and circle your final answers. Justify your answers algebraically whenever possible. Good luck!

1. (5 points) Let $f(x) = (x^2 + 3x + 1)^4(x^3 - 3x)^5$. Find $f'(x)$.

$$\begin{aligned} f'(x) &= (x^2 + 3x + 1)^4 \frac{d}{dx} [(x^3 - 3x)^5] + (x^3 - 3x)^5 \frac{d}{dx} [(x^2 + 3x + 1)^4] \\ &= (x^2 + 3x + 1)^4 \cdot 5(x^3 - 3x)^4 \cdot \frac{d}{dx} (x^3 - 3x) + (x^3 - 3x)^5 \cdot 4(x^2 + 3x + 1)^3 \cdot \frac{d}{dx} (x^2 + 3x + 1) \\ &= 5(x^2 + 3x + 1)^4 (x^3 - 3x)^4 (3x^2 - 3) + 4(x^3 - 3x)^5 (x^2 + 3x + 1)^3 (2x + 3) \\ &= (x^2 + 3x + 1)^3 (x^3 - 3x)^4 [5(x^2 + 3x + 1)(3x^2 - 3) + 4(x^3 - 3x)(2x + 3)] \\ &= (x^2 + 3x + 1)^3 (x^3 - 3x)^4 [23x^4 + 57x^3 - 24x^2 - 81x - 15] \end{aligned}$$

2. (5 points) Given the equation:

$$2x^3 + x^2y - xy^3 = 2,$$

find an expression for y' .

$$\begin{aligned} \frac{d}{dx}(2x^3 + x^2y - xy^3) &= \frac{d}{dx}(2) \\ 6x^2 + \frac{d}{dx}(x^2y) - \frac{d}{dx}(xy^3) &= 0 \\ 6x^2 + x^2y' + y(2x) - (3xy^2 \cdot y' + y^3) &= 0 \\ x^2y' - 3xy^2y' &= y^3 - 6x^2 - 2xy \\ y'(x^2 - 3xy^2) &= y^3 - 6x^2 - 2xy \\ y' &= \frac{y^3 - 6x^2 - 2xy}{x^2 - 3xy^2} \end{aligned}$$

3. (5 points) Differentiate $g(u) = \frac{\ln u}{1 + \ln 2u}$.

$$\begin{aligned} g'(u) &= \frac{(1 + \ln 2u) \cdot \frac{d}{du}(\ln u) - \ln u \cdot \frac{d}{du}(1 + \ln 2u)}{(1 + \ln 2u)^2} \\ &= \frac{(1 + \ln 2u) \cdot \frac{1}{u} - \ln u \cdot (\frac{1}{2u} \cdot 2)}{(1 + \ln 2u)^2} \\ &= \frac{\frac{1}{u} (1 + \ln 2u - \ln u)}{(1 + \ln 2u)^2} = \frac{(1 + \ln 2 + \cancel{\ln u} - \cancel{\ln u})}{u(1 + \ln 2u)^2} \end{aligned}$$

$$g'(u) = \frac{1 + \ln 2}{u(1 + \ln 2u)^2}$$