1. (5 points) Consider $f(x) = 5 - 12x + 3x^2$ on the interval $[1, 3]$. Verify that f satisfies Rolle's Theorem on this interval and find the value c that satisfies the conclusion of Rolle's Theorem.

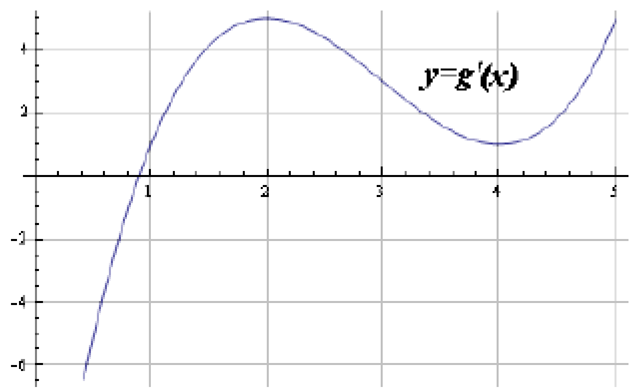
1. f is continuous on $[1,3]$ since f is a polynomial.
2. f is differentiable on $(1,3)$ since f is a polynomial.
3. $f(1) = 5 - 12 + 3 = -4$, $f(3) = 5 - 36 + 27 = -4$, so $f(1) = f(3)$.

By Rolle's Thm, there is $c \in (1,3)$ s.t. $f'(c) = 0$

$f'(x) = -12 + 6x$

$f'(c) = -12 + 6c = 0 \Rightarrow c = 2$

2. Suppose the following is a graph of *the derivative of* a function $g(x)$.



(a) (2 points) On which interval(s) is the function $g(x)$ increasing?

g is increasing when $g' > 0$. So, g is increasing on $(0.9, 5)$

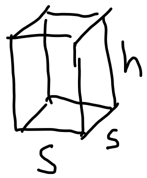
(b) (2 points) On which interval(s) is the graph of $g(x)$ concave down?

g is concave down when $g'' < 0$
(i.e. when g' is decreasing) $\rightarrow$ $(2, 4)$

(c) (1 point) Describe the behavior of g near the point $x = 0.9$.

$g'(x) = 0$ at $x = 0.9$, so 0.9 is a critical point
In particular, $x = 0.9$ is a local minimum of g.

3. (5 points) A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.



$$V = s^2 h$$

$$s^2 h = 32{,}000$$

$$h = \frac{32000}{s^2}$$

$$SA = s^2 + 4sh$$

$$SA = s^2 + 4s\left(\frac{32000}{s^2}\right)$$

call this $A(s)$

$$A(s) = s^2 + \frac{128000}{s}$$

$$A'(s) = 2s - \frac{128000}{s^2}$$

$$0 = 2s - \frac{128000}{s^2}$$

$$\frac{128000}{s^2} = 2s$$

$$s^3 = 64000$$

$$s = 40 \implies h = \frac{32000}{(40)^2} = 20$$

Dimensions are $40 \times 40 \times 20$ cm