MTH 207 – Calculus I
Quiz 7
November 14, 2008

Name: _____
Section: _____

1. (3 points) Use an R_4 sum (four right-handed rectangles) to approximate the area under the graph of $f(x) = x^3 - 2$ between 0 and 4.

$R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x \longrightarrow = f(1)+f(2)+f(3)+f(4)$

$= -1 + 6 + 25 + 62 = \boxed{92}$

$\Delta x = \frac{4-0}{4} = 1$; $x_i = a + i\,\Delta x = i$

2. (4 points) Write the following limit of sums as definite integral on the interval $[0,1]$:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} x_i \ln(1+x_i^2)$$

$$\int_0^1 x \ln(1+x^2)\,dx$$

3. (4 points) Write the following integral as a limit of sums:

$$\int_1^4 (x^2 + 2x - 5)\,dx$$

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$

$x_i = a + i\,\Delta x = 1 + \frac{3i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} (x_i^2 + 2x_i - 5)\left(\frac{3}{n}\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{3i}{n}\right)^2 + 2\left(1+\frac{3i}{n}\right) - 5\right]\left(\frac{3}{n}\right)$

4. (4 points) Evaluate:

$$\int_1^2 \frac{4+u^2}{u^3}\,du$$

$\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 \frac{4}{u^3} + \frac{u^2}{u^3}\,du$

$= \int_1^2 4u^{-3} + \frac{1}{u}\,du$

$= -2u^{-2} + \ln|u| \Big|_1^2$

$= \left(-2(2^{-2}) + \ln 2\right) - \left(-2(1^{-2}) + \ln 1\right)$

$= -\frac{1}{2} + \ln 2 + 2 + \ln 1$

$= \boxed{\frac{3}{2} + \ln 2}$