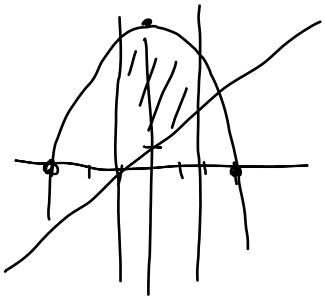
MTH 207 – Calculus I
Quiz 9
December 2, 2008

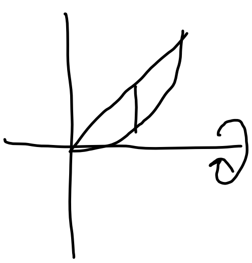
Name: ____________________
Section: ________

1. (5 points) Find the area of the region enclosed by $y = x + 1$, $y = 9 - x^2$, $x = -1$, and $x = 2$.



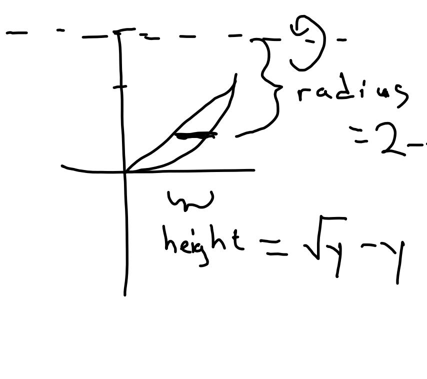
$$A = \int_{-1}^{2} (9 - x^2) - (x+1)\,dx$$
$$= \int_{-1}^{2} 8 - x - x^2\,dx$$
$$= 8x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_{-1}^{2} = \left(16 - 2 - \frac{8}{3}\right) - \left(-8 - \frac{1}{2} + \frac{1}{3}\right)$$
$$= (16 - 2 + 8) - \left(\frac{8}{3} - \frac{1}{3}\right) + \frac{1}{2}$$
$$= 22 - 3 + \frac{1}{2} = \frac{39}{2}$$

2. (5 points) Use the washer method to find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the x-axis.



$$V = \int_0^1 A(x)\,dx$$
$$A(x) = \pi(\text{outside}^2 - \text{inside}^2)$$
$$= \pi(x^2 - (x^2)^2) = \pi(x^2 - x^4)$$
$$V = \pi \int_0^1 x^2 - x^4\,dx$$
$$= \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$$

3. (5 points) Use the shell method to find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the line $y = 2$.



$$V = \int_0^1 2\pi(\text{radius})(\text{height})\,dy$$
$$= \int_0^1 2\pi(2 - y)(\sqrt{y} - y)\,dy$$
$$= 2\pi \int_0^1 2y^{1/2} - 2y - y^{3/2} + y^2\,dy$$
$$= 2\pi\left(\frac{4}{3}y^{3/2} - y^2 - \frac{2}{5}y^{5/2} + \frac{y^3}{3}\right)\Big|_0^1$$
$$= 2\pi\left(\frac{4}{3} - 1 - \frac{2}{5} + \frac{1}{3}\right) = 2\pi\left(\frac{5}{3} - \frac{7}{5}\right) = \frac{8\pi}{15}$$