

The First Calculus Competition of UW-La Crosse

Date: 11/3/12

Name _____

Score _____

Directions: Each problem is worth 10 points. Please show your work on the answer sheets to get full credit.

1. Calculate $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{\sin x}}$.
2. Calculate $\lim_{x \rightarrow 0} \left(\frac{1+x}{\sin x} - \frac{1}{x} \right)$.
3. If $f(x) = (e^x - 1)(e^{2x} - 2)(e^{3x} - 3) \cdots (e^{nx} - n)$, where n is a positive integer, find $f'(0)$.
4. If $f(x) = \begin{cases} \sin[a(x-1)] & \text{if } x \leq 1 \\ \ln x + b & \text{if } x \geq 1 \end{cases}$, find the values of a and b such that $f(x)$ is differentiable at $x = 1$.
5. Given implicit function $y = y(x)$ determined by $x^2 - y + 1 = e^y$, find $\frac{dy}{dx}$.
6. Prove that $\left(1 + \frac{1}{x}\right)^x < e$ for any $x > 0$.
7. The function $f(x)$ is continuous on $[0, 3]$ and differentiable on $(0, 3)$. If $f(0) + f(1) + f(2) = 3$ and $f(3) = 1$, prove that there must exist $\xi \in (0, 3)$ such that $f'(\xi) = 0$.
8. Calculate $\lim_{n \rightarrow \infty} n \left(\frac{1}{1+n^2} + \frac{1}{2^2+n^2} + \cdots + \frac{1}{n^2+n^2} \right)$.
9. Calculate the indefinite integrals $\int \left(1 - \frac{1}{x^2}\right) \sqrt{x\sqrt{x}} \, dx$ and $\int \frac{x}{1+x^4} \, dx$.
10. Calculate the definite integral $\int_0^2 x\sqrt{2x-x^2} \, dx$.