

The Second Calculus Competition of UW-La Crosse

Date: 11/9/13

Name _____

Score _____

Directions: Each problem is worth 10 points. Please show your work on the answer sheets to get full credit.

1. Given $\lim_{x \rightarrow 0} \frac{x - \arctan x}{x^k} = c$, where c and k are constants and $c \neq 0$. Find the values of c and k .
2. Calculate $\lim_{x \rightarrow 0^+} \left(2 - \frac{\ln(1+x)}{x} \right)^{\frac{1}{x}}$
3. Given implicit function $y = f(x)$ determined by the equation $y - x = e^{x(1-y)}$, find the limit $\lim_{n \rightarrow \infty} n \left[f\left(\frac{1}{n}\right) - 1 \right]$.
4. Find an equation of the tangent line to the curve $\sin(xy) + \ln(y-x) = x$ at the point $(0, 1)$.
5. Suppose f is a function that satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y . Suppose also that $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$. Find $f(0)$, $f'(0)$ and $f'(x)$.
6. The odd function $f(x)$ has the second derivative on $[-1, 1]$. If $f(1) = 1$, prove the following statements.
 - (a) There must exist $\xi \in (0, 1)$ such that $f'(\xi) = 1$.
 - (b) There must exist $\eta \in (-1, 1)$ such that $f''(\eta) + f'(\eta) = 1$.
7. Given $f(x) = x^{2x}$, find the minimum value of f on the interval $(0, 1]$.
8. Calculate $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n!}}{n}$.
9. Given $\int_{-\infty}^{\infty} e^{k|x|} dx = 1$, find the value of the constant k .
10. Calculate the integral $\int_0^{\pi} \sqrt{x} \cos \sqrt{x} dx$.
11. Calculate the integral $\int_1^{\infty} \frac{\ln x}{(1+x)^2} dx$.