The Third Calculus Competition of UW-La Crosse

Date: 11/8/14

Name ________________________________ **Score**__________

Directions: Each problem is worth 10 points. Please show your work on the answer sheets to get full credit.

1. Calculate the limit $\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]$.

2. Calculate the limit $\lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)^{1/x^2}$.

3. Given $f(x)=\begin{cases}\ln(x+e) & x>0\\ a^x & x\le 0\end{cases}$, where $a>0$. Determine the value of a such that $f'(0)$ exists.

4. If $f'(a)=1$, find $\lim_{x\to 0}\frac{f(a-3x)-f(a)}{2x}$.

5. Suppose f is an odd periodic function with period 4. If f is differentiable and $f'(x)=2(x-1)$ on $[0,2]$, show your work to find $f(7)$.

6. Suppose that the implicit function $y=f(x)$ is determined by the equation $f(xy)=f(x)+f(y)$, where $f(0)=0$ and $f'(x)\neq 0$. Find $\left.\frac{dy}{dx}\right|_{x=0}$.

7. Suppose $f(x)$ is differentiable on $[0,\infty)$ and $f(0)=0$, if $\lim_{x\to\infty}f(x)=2$, prove the following statements.

   (a) There must exist $a>0$ such that $f(a)=1$.

   (b) For the above a, there exists $\xi\in(0,a)$ such that $f'(\xi)=\frac{1}{a}$.

8. Suppose $f(x)$ and $g(x)$ are both continuous on $[a,b]$, and $f(x)$ is an increasing function. If $0\le g(x)\le 1$, then prove the following statements.

   (a) $0\le\int_a^x g(t)dt\le x-a$, $x\in[a,b]$

   (b) $\int_a^{a+\int_a^b g(t)dt} f(x)dx\le\int_a^b f(x)g(x)dx$

9. Calculate the integral $\int\frac{\sin x\cos x}{1+\sin^4 x}\,dx$.

10. Calculate the integral $\int_{-\infty}^{1}\frac{1}{x^2+2x+5}\,dx$.