

The Fifth Calculus Competition of UW-La Crosse

Date: 11/11/16

Name _____

Score _____

Directions: Each problem is worth 10 points. Please show your work on the answer sheets to get full credit.

1. Calculate the limit: $\lim_{x \rightarrow 0} (\cos 2x + 2x \sin x)^{1/x^2}$.

2. Calculate the limit: $\lim_{x \rightarrow 0} \frac{\int_0^x t \ln(1 + t \sin t) dt}{1 - \cos x^2}$.

3. If $f(x) = \arctan x - \frac{x}{1 + ax^2}$ and $f'''(0) = 1$, find the value of a .

4. Given $f(x) = \begin{cases} x, & x \leq 0 \\ \frac{1}{n}, & \frac{1}{n+1} < x \leq \frac{1}{n} \end{cases}$, where n is any natural number. Does $f'(0)$ exist? Justify your answer.

5. f is continuous on $(-\infty, \infty)$ and $f(x) = (x+1)^2 + 2 \int_0^x f(t) dt$. If $n \geq 2$, find $f^{(n)}(0)$.

6. Calculate the limit $\lim_{x \rightarrow \infty} \frac{1}{n^2} \left(\sin \frac{1}{n} + 2 \sin \frac{2}{n} + \cdots + n \sin \frac{n}{n} \right)$.

7. Given $f(x) = \int_0^1 |t^2 - x^2| dt$, where $x > 0$. Find $f'(x)$ and the minimum value of $f(x)$.

8. The region D is enclosed by $y = \sqrt{1-x^2}$ where $0 \leq x \leq 1$ and $x = \cos^3 t$, $y = \sin^3 t$, where $0 \leq t \leq \pi/2$. Find the volume of the solid that is obtained by rotating D about the x -axis.

9. Calculate

$$\int_1^9 \frac{(2\sqrt{x} + 1)\sqrt{1 + \sqrt{x}}}{\sqrt{x}} dx$$

10. Calculate

$$\int_0^{\pi^{2/3}} \sqrt{x} \sin^2(x^{3/2}) \cos^3(x^{3/2}) dx$$