The Sixth Calculus Competition of UW-La Crosse

Date: 11/11/17

Name _____ **Score**_____

Directions:

Please show your work on the answer sheets to get full credit.

1. Calculate the limit $\lim\limits_{x\to 0^+} \dfrac{\int_0^x \sqrt{x-t}\, e^t\, dt}{\sqrt{x^3}}$.

2. Given $f(x) = \begin{cases} \dfrac{1-\cos\sqrt{x}}{ax}, & x > 0 \\ b, & x \le 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, find the value of ab.

3. Assume $f''(x)$ exists on $(0,1)$ and $f(1) > 0$, $\lim\limits_{x\to 0^+} \dfrac{f(x)}{x} < 0$. Prove the following statements:

   (a) The equation $f(x) = 0$ has at least one solution in the interval $(0,1)$.

   (b) The equation $f(x)f''(x) + (f'(x))^2 = 0$ has at least two distinct solutions in the interval $(0,1)$.

4. If the function $y = f(x)$ is determined by the equation $x^3 + y^3 - 3x + 3y - 2 = 0$, find the extreme values of $f(x)$.

5. Suppose $y(x)$ is differentiable on $(0, \dfrac{3}{2})$ and $y(1) = 0$. Let P be any point on the curve $L:\ y = y(x)$. Let T be the tangent line to L at P and N be the normal line to L at P. If the y-intercept of T equals the x-intercept of N, find an equation for the function $y(x)$.

6. Calculate the limit $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{k}{n^2} \ln(1 + \dfrac{k}{n})$.

7. If the equation $\dfrac{1}{\ln(1+x)} - \dfrac{1}{x} = k$ has a solution in the interval $(0,1)$, find the range of k.

8. Calculate
$$\lim_{t\to\infty} \int_0^t \frac{\ln(1+x)}{(1+x)^2} dx$$

9. Calculate
$$\int_{-\pi}^{\pi} \left(\sin^3 x + \sqrt{\pi^2 - x^2}\right) dx$$