

The Fourth Calculus Competition of UW-La Crosse

Date: 11/7/15

Name _____

Score _____

Directions: Each problem is worth 10 points. Please show your work on the answer sheets to get full credit.

1. Calculate the limit $\lim_{x \rightarrow 0} \frac{\ln \cos x}{x^2}$.
2. Given $\lim_{x \rightarrow 0} \frac{x + a \ln(1+x) + bx \sin x}{kx^3} = 1$, where a , b , and k are constants. Find the values of a , b , and k .
3. If function $f(x) = \lim_{t \rightarrow 0} \left(1 + \frac{\sin t}{x}\right)^{\frac{x^2}{t}}$. Is f continuous on $(-\infty, \infty)$? Justify your answer.
4. Given $f(x) = \begin{cases} x^\alpha \cos \frac{1}{x^\beta} & x > 0 \\ 0 & x \leq 0 \end{cases}$, where $\alpha > 0$ and $\beta > 0$. If $f'(x)$ is continuous at $x = 0$, what is your conclusion on the value of $\alpha - \beta$?
5. Given $f(x) = x^2 \cdot 2^x$. Find $f^{(n)}(0)$.
6. Suppose $f(a) = 0$ and both of $f'(x)$ and $f''(x)$ are positive on $[a, \infty)$. Assume $b > a$ and $P(b, f(b))$ is a point on the curve $y = f(x)$. If the tangent line to $y = f(x)$ at P has the x -intercept $(x_0, 0)$, prove that $a < x_0 < b$.
7. Suppose $f(x)$ is continuous and $\phi(x) = \int_0^{x^2} xf(t)dt$. If $\phi(1) = 1$ and $\phi'(1) = 5$, find $f(1)$.
8. Suppose $f(x) = \int_0^x e^{-u} \cos u \, du$. Find the absolute minimum and maximum values of f on the interval $[0, \pi]$.
9. Calculate the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\sin x}{1 + \cos x} + |x| \right) dx$.
10. Calculate the integral $\int \frac{dx}{(1 + e^x)^2}$.