Math 150

Semester Review: Chapter 1

1. Which would you use ($<$, $>$, or $=$) to make the statement true?
   a) $\frac{2}{7}$ ___ 0.2857

   b) $2\sqrt{3}$ ___ $\frac{2\pi}{3}$

2) The coordinate of A and B, respectively, on a coordinate line are x and 6. Find the distance $d(A, B)$.

3) Rewrite the expression without using the absolute value symbol and simplify the result.
   a) $|5 - x|$ if $x < 0$.

   b) $\left|\frac{10}{3} - \pi\right|$

Perform indicated operations and simplify.

4) $\left(\frac{4x^3 y^{-1}}{z^2}\right)^{-3}$

5) $\frac{(2x)^{-4}(xy)^{-3}}{y^{-2}}$

6) $\frac{(2x^3)(3y^2)^2}{(y^3)^4} \cdot (x^3)^0$

7) $(3r^3 s)^2 \cdot (-2r^{-1} s^4)^2$

8) $\sqrt[5]{\frac{3}{16x^3}}$

9) $\frac{4x}{1-\sqrt{2}}$, rationalize the denominator.

10) $\left(\frac{c^{-3}}{8d^6}\right)^{4/3}$

11) $-27^{-\frac{2}{3}}$

Express as a simplified polynomial.

12) $(-3x^3 + 7x^2 - x + 1) - (4x^3 - 2x - 5)$

13) $(2x - 4)^3$

14) $(5x + 1)(x - 2) - (x - 8)$

15) $\dfrac{4a^4b^3 - 12a^3b^2 + 16a^2b^2}{4a^2b}$

Factor each completely.

16) $2x^3 - x^2 + 10x - 5$

17) $2t^3 - 128$

18) $81x^4 - 1$

19) $x^2 - x - 56$

20) $3x^3 + 18x^2 + 27x$

Perform the indicated operation and simplify.

21) $\dfrac{2x^2 + x - 3}{x^2 + 7x + 12} \cdot \dfrac{x^3 + 5x^2 + 6x}{4x^2 + 6x}$

22) $\dfrac{x^3 - 1}{x^2 - 1} \div \dfrac{x^2 + x + 1}{x^3 + 1}$

23) $\dfrac{-2x}{x-4} - \dfrac{3}{2x+5}$, also, find the domain.

Simplify each of the following.

24) $\dfrac{\frac{3}{x}+2}{2-\frac{1}{x+1}}$

25) $\dfrac{x+y}{x^{-1}y - xy^{-1}}$

26) $\dfrac{x(x+2)^{-1/3}-(x+2)^{2/3}}{x^2}$

27) $x^3(2-x^2)^{-1/4} - 3x(2-x^2)^{3/4}$

28. $\sqrt[3]{-125x^6y^4z^{15}}$

29. $\sqrt[4]{32a^5b^8}$

Semester Review: Chapter 2

2.1
Solve each equation.

1. $\frac{5}{3}x - 1 = 4 + \frac{2}{3}x$

2. $\frac{-5}{3x-9} + \frac{4}{x-3} = \frac{5}{6}$

3. $\frac{3}{2x+5} + \frac{4}{2x-5} = \frac{14x+3}{4x^2-25}$

4. Solve $F = g \cdot \frac{mM}{d^2}$ for m.

2.2

5. Show a chart and all your work to solve the following:
A pharmacist is to prepare 15 milliliters of special eye drops for a glaucoma patient. The eye-drop solution must have a 2% active ingredient, but the pharmacist only hs 10% and 1% solution in stock. How much of each type of solution should be used to fill the prescription?

6. A student in Math 150 has test scores of 74, 73, 69, and 80. What score on the next test will raise the student's average to 75?

7. A runner starts at the beginning of a runners' path and runs at a constant rate of 6 mi./hr. Five minutes later a second runner begins at the same point, running at a rate of 8 mi./hr and following the same course. How long will it take the second runner to reach the first?

2.3

8. By factoring, solve $15x^2 + 14 = 29x$.

9. By factoring, solve $\frac{5x}{x-2} + \frac{3}{x} + 2 = \frac{-6}{x^2 - 2x}$.

10. Solve $16x^2 = 49$.

11. Solve $9(x-1)^2 = 7$.

12. By completing the square, solve $x^2 - 8x + 11 = 0$.

13. By using the quadratic formula, solve $3x^2 + 5x + 1 = 0$.

14. Solve $F = g \cdot \frac{mM}{d^2}$ for d.

2.4

For #15-19, simplify each and write in the form $a + bi$.

15. $(-3 + 8i) - (2 + 3i)$

16. $(6 + 7i)^2$

17. i^{66}

18. $\frac{-3-2i}{5+2i}$

19. $(-3+\sqrt{-25})(8 - \sqrt{-36})$

20. Solve $4x^2 + x + 3 = 0$

2.5

For #21-25, solve each.

21. $\sqrt[3]{6 - s^2} + 5 = 0$

22. $2|5x + 2| - 1 = 5$

23. $\sqrt{7 - x} = x - 5$

24. $2x^4 - 10x^2 + 8 = 0$

25. $x^{4/3} = 16$

2.6
Solve the inequalities, and express the solutions in terms of intervals whenever possible.

26. $-2 < \frac{2x-1}{6} < 5$

27. $|3x - 5| \geq 6$

28. $|6x - 5| < 19$

2.7

Solve the inequalities, and express the solutions in terms of intervals whenever possible.

29. $(x+3)(x-5)(-2-x) \leq 0$

30. $\dfrac{(x^2+1)(x-3)}{x^2-9} \geq 0$

Semester Review: Chapter 3

3.1

1. Given $A(-5, 9)$ and $B(-8, -7)$ find:

    a. the distance $d(A, B)$

    b. the midpoint of segment AB

    c. the slope of line AB

2. Find the equation of the circle that has center $C(7, -4)$ and passes through $P(-3, 3)$.

3. Show that the points $A(2, 1)$, $B(4, 0)$, and $C(5, 7)$ are the vertices of a right triangle.

3.2

4. Find the x-intercept(s), y-intercept(s), then test for all types of symmetry, and sketch the

  graph of the equation.

    a. $x = 2y^2 - 4$ b. $y = -x^3 + 1$
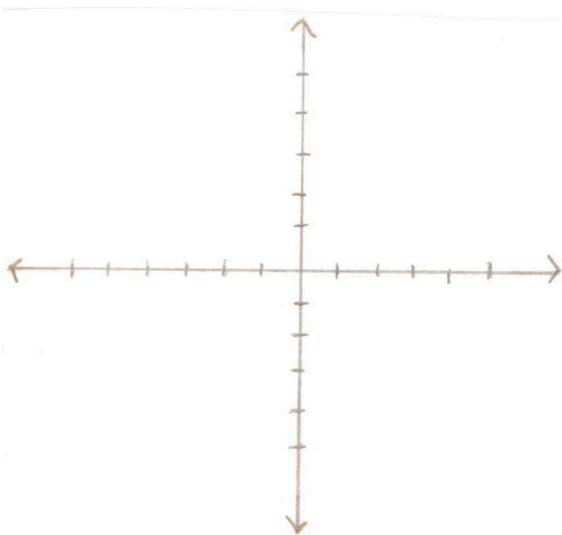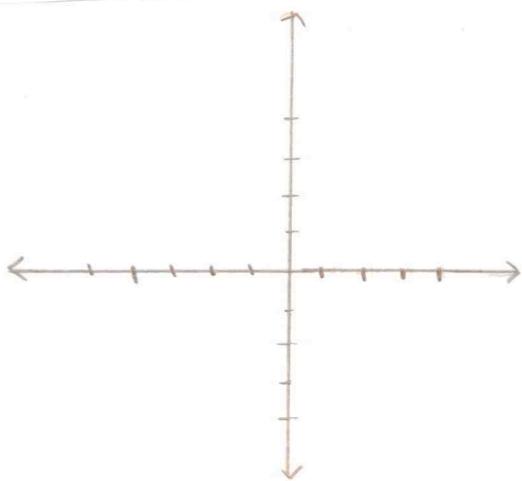
5. Find the equation for a circle with center $(-1, 2)$ and contains the point $(3, 4)$.

6. Find the standard form for a circle with the following equation. Then list its center and radius. $4x^2 + 4y^2 + 20x - 16y + 37 = 0$

3.3

7. Find the general form equation of a line with x-intercept 6 and y-intercept -3.

8. What is the slope of a line perpendicular to $2x - 3y = 5$?

9. What is the slope of a line parallel to $y = 5$?

10. Find the equation, in gerneral form, of a line parallel to $y = \frac{1}{5}x + 8$ and passing through the point $(3, 1)$.

3.4

11. If $g(x) = 4x - x^2$, find:

    a. $g(2)$ b. $g(x + 2)$

12. If $f(x) = -2x + 3$, find:

    a. $f(x + h)$ b. $\frac{f(x+h) - f(x)}{h}$

13. Find the domain of $f(x) = \frac{\sqrt{8 - 3x}}{x^2 - 1}$

3.5

14. Determine if $f(x)$ is even, odd, or neither. Show your work that verifies your answer.

    a. $f(x) = \frac{x^3}{4}$ b. $f(x) = 3x^2 - 5x + 1$

15. Explain how the graph of the function compares to the graph of $y = f(x)$.

    a. $y = f(\frac{1}{2}x) - 3$ b. $y = -f(x + 4) - 2$

16. The graph of $f(x)$ is given. Sketch the graph of:
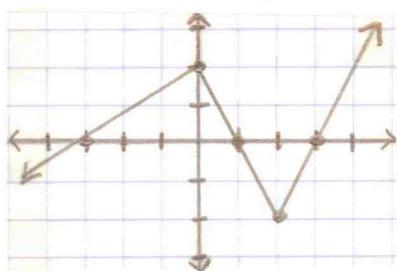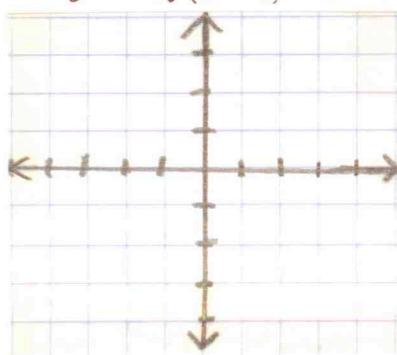
    a. $y = -2f(x)$
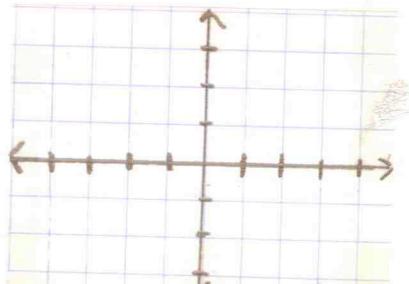
    b. $y = -f(x + 1) - 3$ c. $y = |f(x)|$
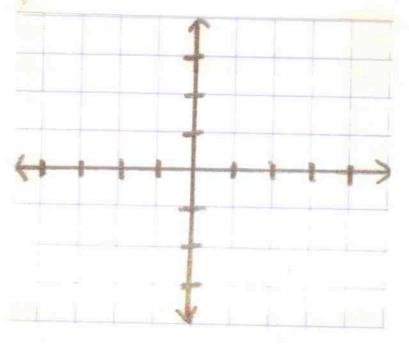
17. Sketch the graph of $f(x)$. Then state the domain and range.

$$f(x) = \begin{cases} x+2 & if\ x \leq -1 \\ x^3 & if\ -1 < x < 1 \\ |x| & if\ x \geq 1 \end{cases}$$
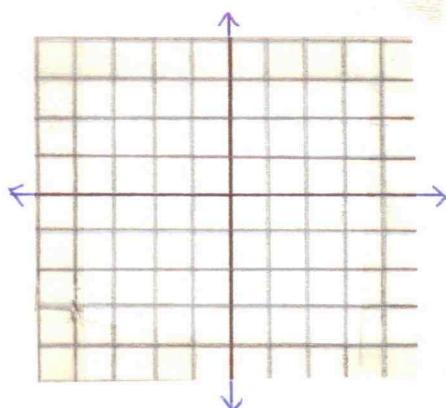
3.6

18. If $f(x) = 2x^2 + 8x + 7$, find:

    a. Standard form of the quadratic equation.

    b. The vertex.

    c. Does it have a maximum or minimum?

    d. What is the maximum or minimum?

19. Find the standard equation for a parabola that has a vertical axis, the vertex $(0, 5)$, and passes through $(2, -3)$.

20. Find the maximum or minimum value of $f(x)$. Which is it, a max or min?
    $$f(x) = -4x^2 + 4x - 1$$

3.7

21. If $f(x) = 2x + 1$ and $g(x) = x^2 + 2x + 1$, find:
    a. $(f + g)(x)$ b. $(f - g)(x)$

    c. $\left(\frac{f}{g}\right)(x)$ d. State the domain of $\left(\frac{f}{g}\right)(x)$.

22. Find $(f \circ g)(x)$ and its domain if $f(x) = \sqrt{x}$ and $g(x) = x - 1$.

Semester Review: Chapter 4

4.1

1. Sketch a graph of $f(x) = -2x^3 - 3$.
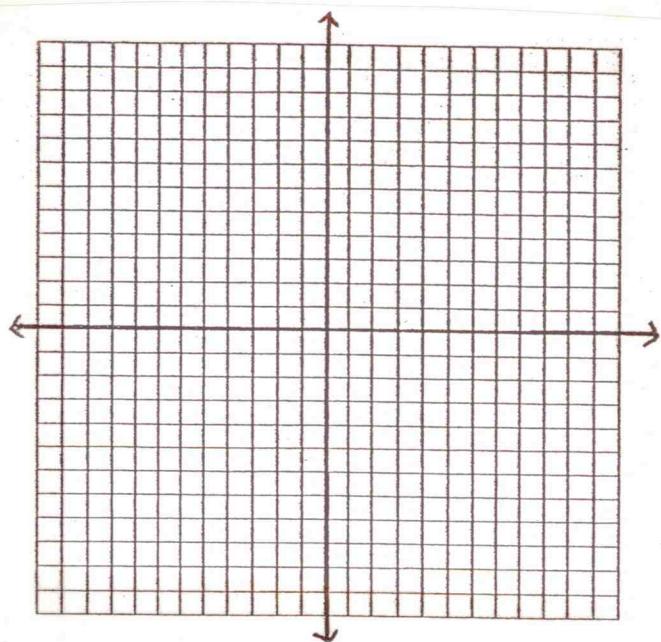
2. Find all values of x such that $f(x) > 0$ and all x such that $f(x) < 0$, and sketch the graph of f. $f(x) = x^4 + 3x^3 - 4x^2$
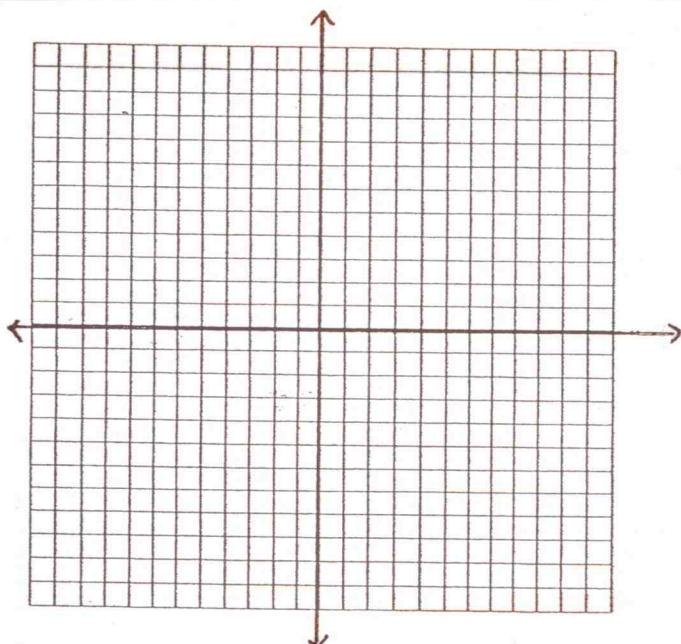
3. Sketch the graph of a polynomial given the following sign diagram.

Sign of $f(x)$

$$+\ \big|\ +\ \big|\ -\ \big|\ +\ \big|\ -$$

$-3\ -2\qquad 0\qquad 2$
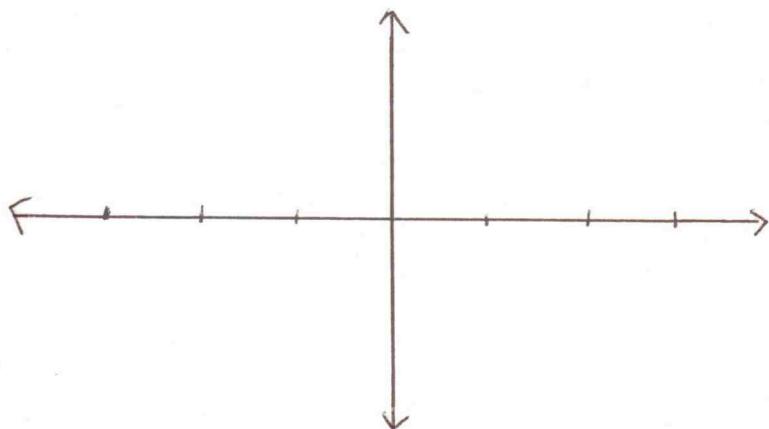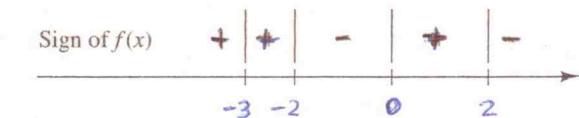
4.2

4. Find the quotient and reaminder if $f(x)$ is divided by $p(x)$.

$$f(x) = 8x^3 - 6x^2 - 4x - 8 \qquad\qquad p(x) = 2x^2 + x$$

5. Use the remainder theorem to find $f(c)$.

$$f(x) = 2x^4 + 3x^3 + x - 2 \qquad\qquad c = -2$$

6. Use synthetic division to find the quotient and remainder if $-x^3 - 4x^2 - x + 8$ is divided by $x + 4$.

7. Find a polynomial $f(x)$ of degree 3 with real coefficients that has $2i$, and 3 as zeros and where $f(1) = 20$.

8. Find a polynomial function, in factored form, of degree 3 whose graph is shown.
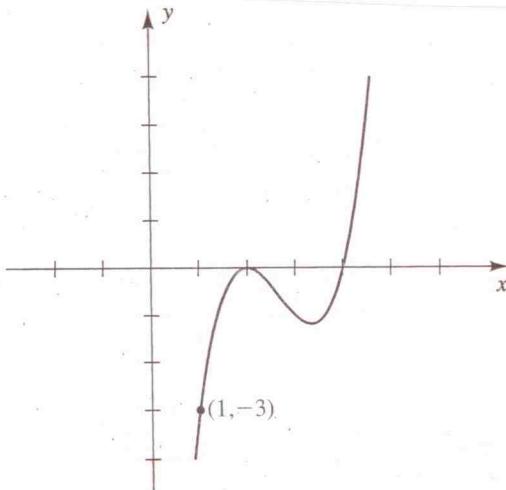
9. Use Decartes' Rule of Signs to determine the number of possible positive, negative, and nonreal complex solutions of the equation.
$$2x^4 - x^3 + x^2 - 3x + 4 = 0$$

10. List all possible rational roots for $2x^3 - 3x^2 - 17x + 15 = 0$.

11. Find all solutions of $f(x) = 6x^4 + 5x^3 - 17x^2 - 6x$.

12. Find all solutions of $f(x) = 2x^3 + 2x^2 + 18x + 18 = 0$.

13. Find the factored form with integer coefficients of polynomial f shown in the figure.

$$f(x) = -6x^5 + 5x^4 + 14x^3 - 8x^2 - 8x + 3$$
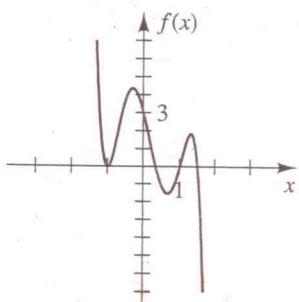
4.5

14. Find the asymptotes (if any exist) for $f(x) = \frac{x^2 - 2x + 4}{x + 1}$.

VA_____

HA_____

Oblique_____

15. Find the asymptotes (if any exist) for $f(x) = \frac{x - 5}{x^2 - 5}$.

VA_____

HA_____

Oblique_____

16. Sketch each graph of f. Make sure to find all x and y-intercepts and asymptotes (vertical, horizontal, and oblique).

a) $f(x) = \frac{x^2 - 2x + 4}{x + 1}$

x-intercept (s)_____

y-intercept _____

VA(s)_____

HA(s)_____
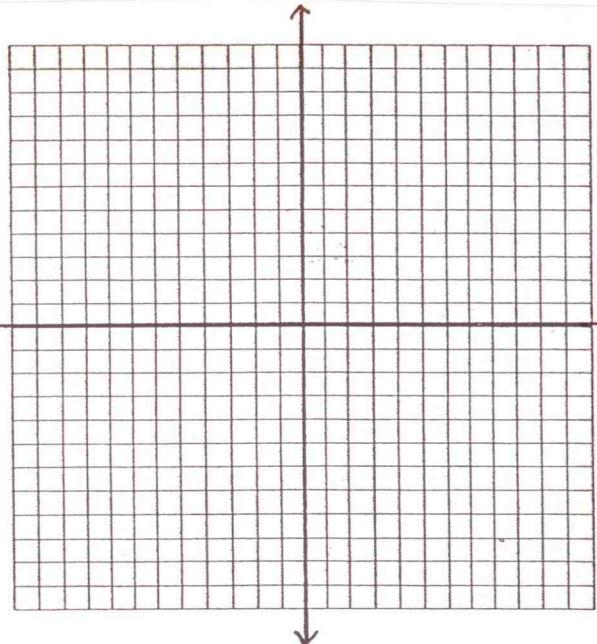
Oblique A(s)_____

b) $f(x) = \frac{2x^2 + 8x + 6}{x^2 - 2x}$

x-intercept(s)_____

y-intercept _____

VA(s)_____

HA(s)_____

Oblique A(s)_____
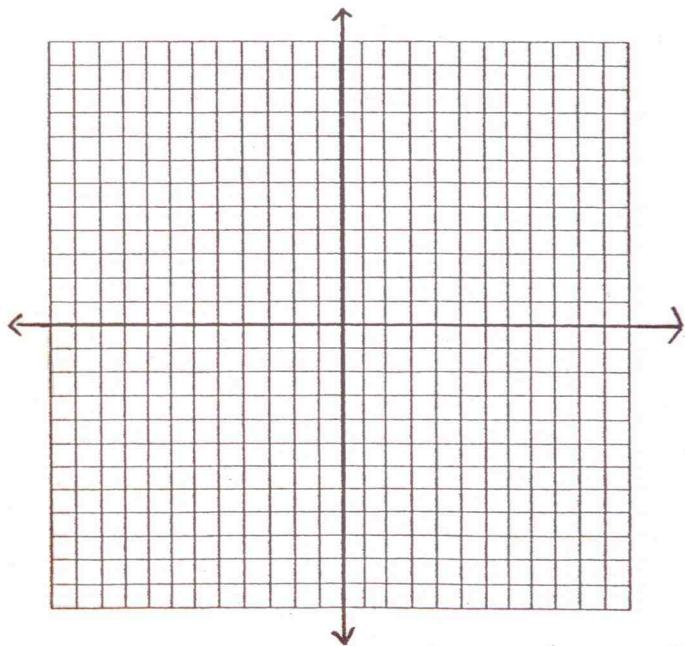
4.6

17. z varies directly as the square root of x and inversely as the square of y. When $x = 100$ and $y = 5$, $z = 4$. Find the constant of variation. Find z when $x = 0.04$ and $y = 3$.

Semester Review: Chapter 5

Show all your work.

5.1

1. Determine whether the function is one-to-one.
$$f(x) = \frac{1}{x^2}$$

2. Verify that $f(x)$ & $g(x)$ are inverse functions.
$$f(x) = \frac{5-3x}{2} \quad g(x) = \frac{5-2x}{3}$$

3. Find the inverse functions of $f(x) = \sqrt{2x-3}, x \geq \frac{3}{2}$.

4.

x	2	3	4	5	6
$f(x)$	-1	0	1	2	3

x	-1	2	3
$g(x)$	1	3	5

Find: a. $(g \circ f^{-1})(-1)$ b. $(f^{-1} \circ g^{-1})(3)$

5.2

5. Solve each of the following:

   a. $6^{7-x} = 6^{2x+1}$ b. $9^{x^2} = 3^{3x+2}$

   c. $\left(\frac{1}{2}\right)^{6-x} = 2$

6. Sketch a graph of $f(x) = 8(4)^{-x} - 2$. State its horizontal asymptote, its x-intercept, and its y-intercept.
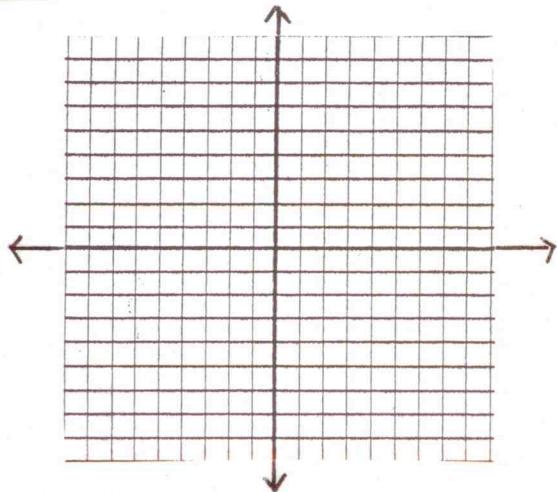
7. Find an exponential function of the form $f(x) = ba^x$ or $f(x) = ba^x + c$ that has the given graph.

$(-2, 8)$

$\left(0, \frac{1}{2}\right)$
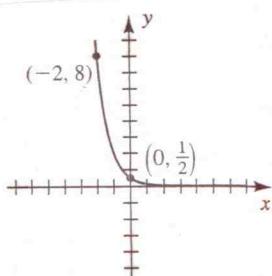
5.3

8. If $10,000$ is deposited into an account that pays a 10.5% rate of interest per year compounded continuously, find the balance after 12 yrs.

9. How much money, invested at an interest rate of 9.5% per year compounded continuously, will amount to $15,000$ after 4 yrs.?

10. Find the zeros of f. $f(x) = -x^2 e^{-x} + 2xe^{-x}$

11. The 1985 population estimate for India was 762 million, and the population has been growing continuously at a rate of about 2.2% per year. Assuming this rapid growth rate continues, estimate the population $N(t)$ of India in the year 2010.

5.4

12. Find the Domain of the function $f(x) = log_3(x+8)$.

13. $y = log_a x$ if and only if ___?___.

14. Change $3^4 = \frac{1}{x}$ to logarithmic form.

15. Change each to exponential form.
    a. $log_6(2x - 1) = 3$ b. $ln\, z = 7 + x$

16. Solve: $log_7(x - 5) = log_7(6x)$.

17. Solve: $e^{2\,ln\,x} = .25$

18. Find the logarithmic function of the form, $f(x) = log_a x$ for the given graph.
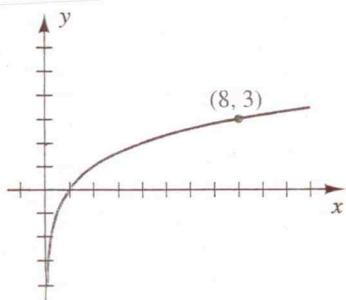
19. Graph $f(x) = -\frac{1}{2}\log(x+1)$. State its vertical asymptote, its doamin and its range.
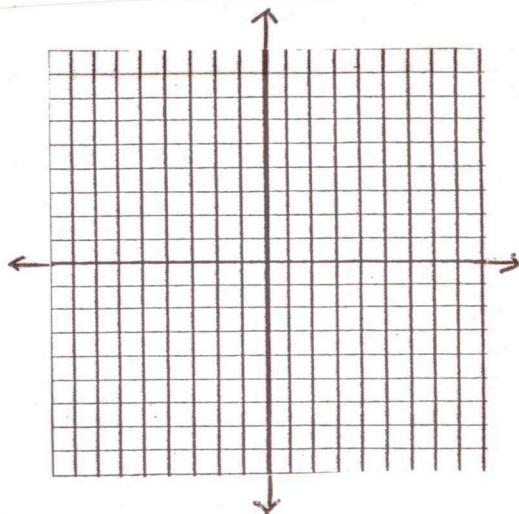
20. Express the logarithmic expression in terms of logarithms of x, y, and z.

$$\log_a\left(\frac{x}{y^5\sqrt{z}}\right)$$

21. Condense the expression to a single logarithm.

$$\log a + 8\log b - \frac{1}{3}\log c$$

22. Solve: $\log_3(x^2 - 4) - \log_3(x - 2) = 4$.

23. Estimate $\log_5 9$ to the nearest hundredths.

24. Solve: $2^{2x-3} = 5^{x-2}$.

25. Solve: $log\,(5x + 1) = 2 + log\,(2x - 3)$.

26. You deposit $12,000 into an account that pays 5% annual interest compounded continuously.
Find the balance at the end of 8 years.
How long will it take your money to triple? $[A = Pe^{rt}]$

Chapter 1 - Answers to Review

1a) $>$

1b) $>$

2) $|x - 6|$

3a) $5 - x$

3b) $\frac{10}{3} - \pi$

4) $\frac{y^3 z^6}{64 x^9}$

5) $\frac{1}{16 x^7 y}$

6) $\frac{18 x^3}{y^8}$

7) $36 r^4 s^{10}$

8) $\frac{\sqrt[5]{6 x^2}}{2x}$

9) $-4x(1 + \sqrt{2})$

10) $\frac{1}{16 c^4 d^8}$

11) $-\frac{1}{9}$

12) $-7x^3 + 7x^2 + x + 6$

13) $8x^3 - 48x^2 + 96x - 64$

14) $5x^2 - 10x + 6$

15) $a^2 b^2 - 3ab + 4b$

16) $(2x - 1)(x^2 + 5)$

17) $2(t - 4)(t^2 + 4t + 16)$

18) $(9x^2 + 1)(3x + 1)(3x - 1)$

19) $(x - 8)(x + 7)$

20) $3x(x + 3)^2$

21) $\frac{(x+2)(x-1)}{2(x+4)}$

22) $x^2 - x + 1$

23) $\frac{-4x^2 - 13x + 12}{(x-4)(2x+5)}$ Domain = All Reals $x \neq 4, -\frac{5}{2}$

24) $\frac{(2x+3)(x+1)}{x(2x+1)}$

25) $\frac{xy}{y-x}$

26) $-\frac{2}{x^2(x+2)^{1/3}}$

27) $\frac{2x(2x^2 - 3)}{(2 - x^2)^{1/4}}$

28) $-5x^2 y z^5 \sqrt[3]{y}$

29) $2ab^2 \sqrt[4]{2a}$

Answers: Semester Review Chapter 2

1. $x = 5$

2. $x = \frac{29}{5}$

3.   No solution

4. $m = \frac{Fd^2}{gM}$

5.

	Amount Solution	%	Amount Active Ingredient
10%	x	.1	$.1x$
1%	$15 - x$	.01	$.01(15 - x) = .15 - .01x$
2%	15	0.02	$.02(15) = .3$

$1.67\,ml$ of 10% solution & $13.33\,ml$ of 1% solution

6. $x = 79$

7. $\frac{1}{4}$ hr or $15\,min.$

8. $x = \frac{14}{15}, 1$

9. $x = \frac{1}{7}$

10. $x = \pm\frac{7}{4}$

11. $x = 1\pm\frac{\sqrt{7}}{3}$

12. $x = 4\pm\sqrt{5}$

13. $x = \frac{-5\pm\sqrt{13}}{6}$

14. $d = \sqrt{\frac{gmM}{F}}$

15. $-5 + 5i$

16. $-13 + 84i$

17. -1

18. $\frac{-19}{29} - \frac{4}{29}i$

19. $6 + 58i$

20. $\frac{-1\pm i\sqrt{47}}{8}$

21. $s = \pm\sqrt{131}$

22. $x = \frac{1}{5},\ -1$

23. $x = 6$

24. $x = 2,\ -2, 1,\ -1$

25. $x = 8$

26. $\left(-\frac{11}{2}, \frac{31}{2}\right)$

27. $\left(-\infty, -\frac{1}{3}\right], \left[\frac{11}{3}, \infty\right)$

28. $\left(-\frac{7}{3}, 4\right)$

29. $[-3, -2], [5, \infty)$

30. $(-3, 3), (3, \infty)$

Answers: Review Ch. 3

1. a. $\sqrt{265}$ b. $\left(-\frac{13}{2}, 1\right)$ c. $\frac{16}{3}$

2. $(x-7)^2 + (y+4)^2 = 149$

3. $(AB)^2 + (AC)^2 = (BC)^2$
$$\sqrt{5}^2 + \sqrt{45}^2 = \sqrt{50}^2$$
$$5 \;+\; 45 \;=\; 50$$
$$50 \;=\; 50$$

4. a) x-int. $= (-4, 0)$ b) x-int. $= (1, 0)$ 4a)
       y-ints. $= (0, \pm\sqrt{2})$ y-int. $= (0, 1)$
       Symmetry about the x-axis   No symmetry

5. $(x+1)^2 + (y-2)^2 = 20$

6. $\left(x+\frac{5}{2}\right)^2 + (y-2)^2 = 1$
Center $= \left(-\frac{5}{2}, 2\right)$; $r = 1$

7. $x - 2y = 6$

8. $-\frac{3}{2}$ 4b)

9. 0

10. $x - 5y = -2$

11. a. 4   b. $-x^2 + 4$

12. a. $-2x - 2h + 3$ b. -2

13. $(-\infty, -1) \cup (-1, 1) \cup \left(1, \frac{8}{3}\right]$

14. a. odd   b. neither

15. a. Horizontal stretch by a factor of 2 and shifted down 3.

   b. Reflected over the x-axis and shifted left 4 and down 2.
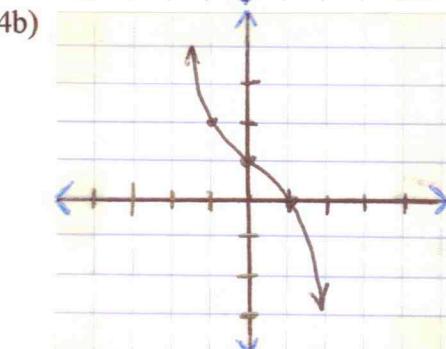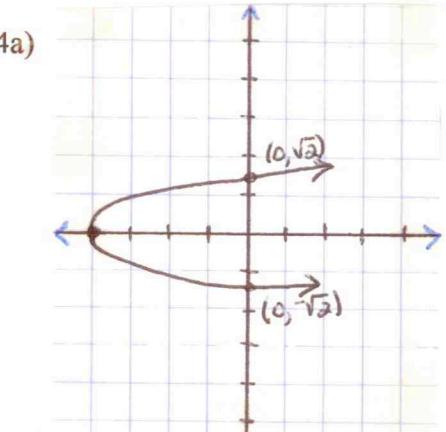
16a)
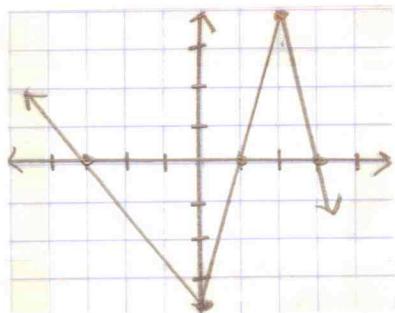
16b)
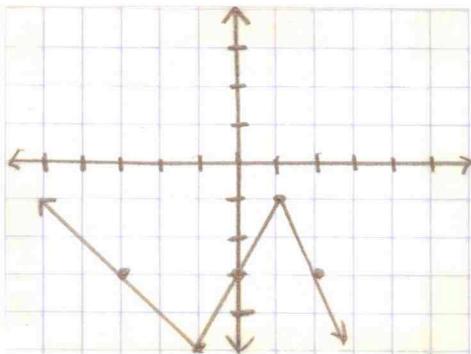
16c)
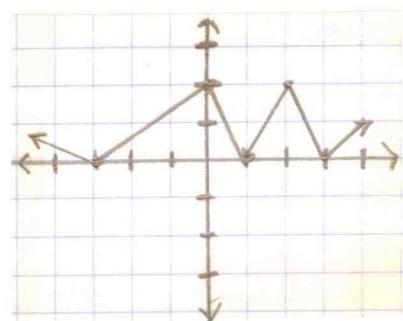
17.
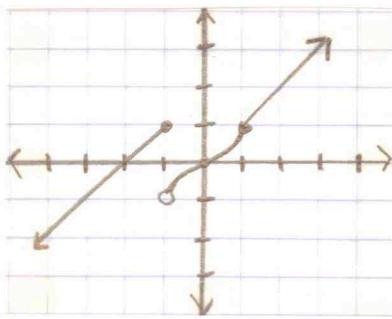
18. a. $f(x) = 2(x+2)^2 - 1$ 　　　　b. $V = (-2, -1)$

　　c. minimum 　　　　　　　　d. Min $= -1$

19. $y = -2x^2 + 5$

20. Max $= 0$

21. a. $x^2 + 4x + 2$ 　　　　　　b. $-x^2$

　　c. $\frac{2x+1}{x^2+2x+1}$ 　　　　　　d. All Reals except $x = -1$

22. $f(g(x)) = \sqrt{x-1}$ 　　$D : [1, \infty)$

Answers: Semester Review Chapter 4

1.
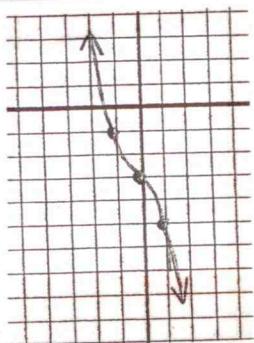
2. $f(x) > 0: \ (-\infty, -4), \ (1, \infty)$
     $f(x) < 0: \ (-4, 0), \ (0, 1)$
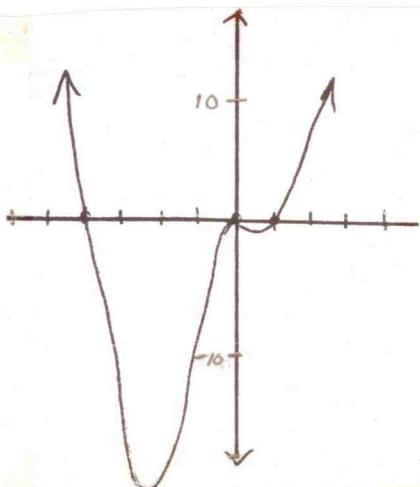
3.
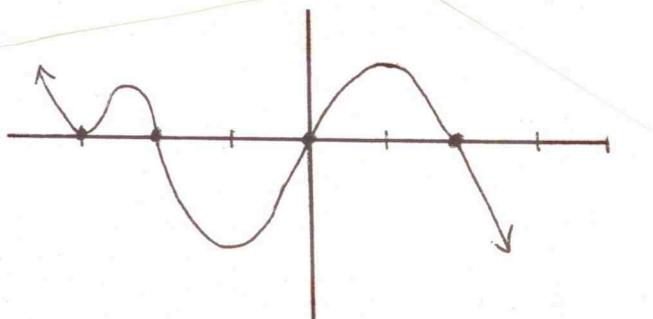
4. $Q(x) = 4x - 5$
     $r(x) = x - 8$

5. $f(-2) = 4$

6. $Q(x) = -x^2 - 1$
     $r(x) = 12$

7. $f(x) = -2x^3 + 6x^2 - 8x + 24$

8. $f(x) = (x-2)^2(x-4)$ $Note: \{a = 1\}$

9.

$+ R's$	$- R's$	$Comp.$
4	0	0
2	0	2
0	0	4

10. $\pm\{1, \frac{1}{2}, 3, \frac{3}{2}, 5, \frac{5}{2}, 15, \frac{15}{2}\}$

11. $x = -\frac{1}{3}, \frac{3}{2}, -2, 0$

12. $-1, 3i, -3i$

13. $f(x) = -(2x-3)(3x-1)(x+1)^2(x-1)$

14. VA: $x = -1$, HA: none, Obl.A: $y = x - 3$

15. VA: $x = \sqrt{5}$, $x = -\sqrt{5}$, HA: $y = 0$
    Obl.A: None

16a. x-int: None
    y-int: $(0, 4)$
    VA(s): $x = -1$
    HA(s): None
    Obl.A: $y = x - 3$
    {See Graph}
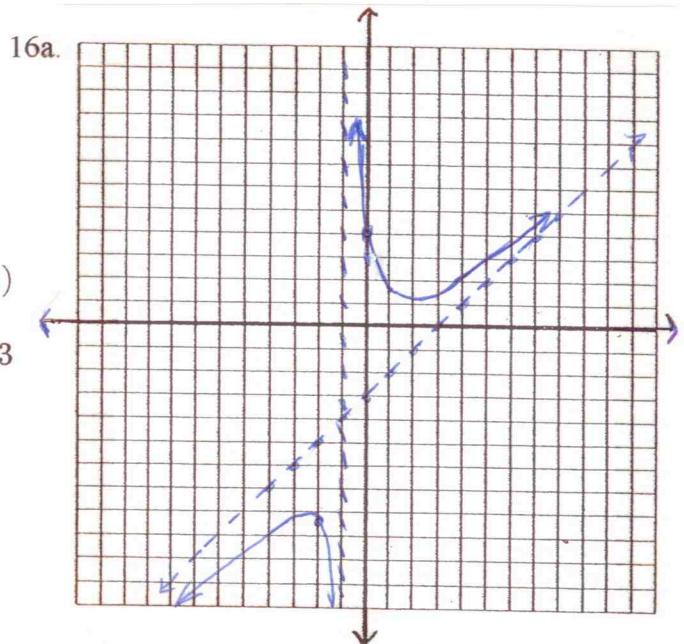
16b. x-int: $(-3, 0)$, $(-1, 0)$
    y-int: None

    VA(s): $x = 0$, $x = 2$
   HA(s): $y = 2$
    Obl.A: None
    {See Graph}

17. $k = 10$
    $z = \frac{2}{9}$

16a.

16b.
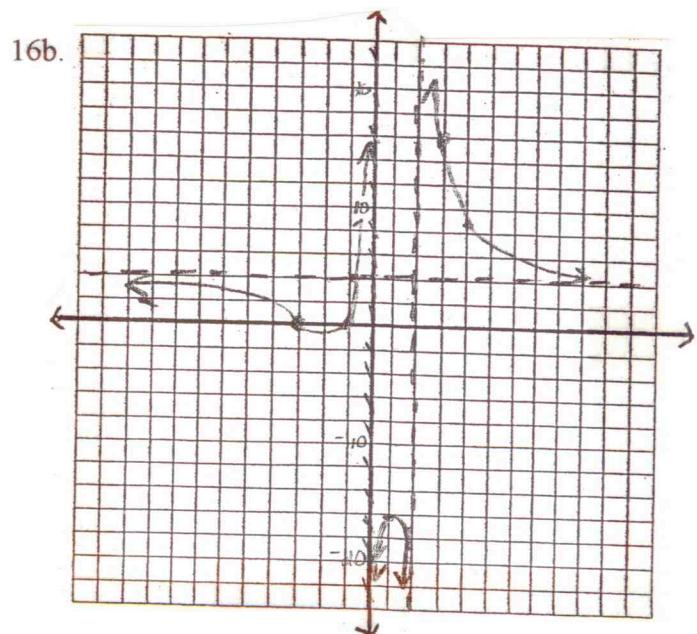
Math 150

1. Show that when $f(a) = f(b)$, $a = \pm b$, so it is not one-to-one.

2. Show $f(g(x)) = x$ and $g(f(x)) = x$.

3. $f^{-1}(x) = \frac{x^2+3}{2}$

4. a) $g(f^{-1}(1)) = g(2) = 3$

   b) $f^{-1}(g^{-1}(3)) = f^{-1}(2) = 5$

5. a) $x = 2$

   b) $x = -\frac{1}{2}, 2$

   c) $x = 7$

6. HA: $y = -2$, x-intercept: $(1, 0)$, y-intercept: $(0, 6)$ {graph}

7. $f(x) = \frac{1}{2}\left(\frac{1}{4}\right)^x$

8. $35,254.21

9. $10,257.92

10. $x = 0, 2$

11. 1320.7 mil

12. $(-8, \infty)$

13. $a^y = x$

14. $\log_3\left(\frac{1}{x}\right) = 4$

15. a) $6^3 = 2x - 1$

    b) $e^{7+x} = z$

16. No Solution
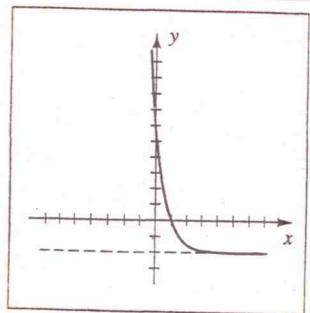
17. $x = \frac{1}{2}$

18. $f(x) = \log_2 x$

19. VA: $x = -1$, D: $(-1, \infty)$, R: $(-\infty, \infty)$ {graph}

20. $\log_a x - 5\log_a y - \frac{1}{2}\log_a z$

21. $\log \frac{ab^8}{\sqrt[3]{c}}$

22. $x = 79$

23. ≈ 1.37

24. $x = \frac{3\log 2 - 2\log 5}{2\log 2 - \log 5}$ or $\frac{\log 8 - \log 25}{\log 4 - \log 5}$ or $\frac{3\ln 2 - 2\ln 5}{2\ln 2 - \ln 5}$ $\frac{\ln 8 - \ln 25}{\ln 4 - \ln 5} \approx 5.11$

25. $x = \frac{301}{195} \approx 1.54$

26. After 8 yrs: \$17,901.90, Triple: $\frac{\ln 3}{0.05} \approx 22$ yrs.