FROM SPINS TO STARS:

Informing Astrophysical Scenarios through Indirect Measurements on Radioactive Nuclei

Kelly Chipps – Oak Ridge National Laboratory
Overview

I) Introduction to Nuclear Astrophysics:
   From the very small to the very large
   (32 orders of magnitude in dimension, 60 in energy):
   how quantum subtleties alter astronomical events,
   and the origin of the elements

II) Connecting Nuclei to the Stars:
   Studying stars in the laboratory,
   with some nuclear reaction examples

III) Improving our Understanding of Nucleosynthesis:
   Better science with better targetry
The Big Questions:
What is the origin of the elements in the cosmos?
What are the nuclear reactions that drive stars and stellar explosions?
Welcome to Your Universe
Sirius is the brightest star in the night sky

In 1844, the German astronomer Friedrich Bessel deduced that Sirius is actually a binary system.

The small, binary companion, Sirius B (the “pup” star), was first observed in 1862 by American astronomer Alvin Clark.

In 1915, observation led to the discovery that Sirius B was a white dwarf – much hotter and denser than its larger companion.
What's so special about a white dwarf?

- White dwarf stars are the very hot and very dense remnants of dead main-sequence stars (the burned-out cores, rich in carbon, oxygen, and neon, and held in shape only by electron degeneracy pressure).
- When they're close to another star, their large gravitational pull can cause material to fall onto the dwarf's surface ("accrete").
What's so special about a white dwarf?

- that accreting material gets hot and dense, too, and then *flash!* a **runaway thermonuclear explosion** on the white dwarf surface – a **nova**
From Sirius, to Mira... to the Universe

About 50% of all stars are in binary systems!
Novae aren't rare (~40/yr) - Sirius B won't become one (the companions are too distant), but the Mira binary system is in the process right now!

x-ray image of accretion! →
Explosive Astrophysical Environments

- other explosive events
  - novae
  - x-ray bursts
  - supernovae
- reactions proceed faster than competing decay
  - final isotopic abundances determined by network of reactions on short-lived, highly unstable nuclei
- these environments are the source of much of the heavy elements in the universe

...and they're the subject of lots of interdisciplinary study!
Astrophysical Nuclear Reactions

→ Maxwell-Boltzmann distribution
  *probability of a particle having a given energy for environmental temperature*

→ Barrier penetrability
  *quantum likelihood of particles getting close enough to react at a given energy*

→ Convolution = “Gamow peak”
  *nuclear levels falling within this energy range will contribute strongly to the stellar reaction rate*
Astrophysical Nuclear Reactions

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  *nuclear levels falling within this energy range will contribute strongly to the stellar reaction rate*

To quantify the contribution from an astrophysical nuclear reaction, we need to measure either:

the **rate** due to each of the levels in the Gamow peak directly, or the **nuclear properties** of these same levels (energies, spins, decay branches, spectroscopic factors) from which the rate can be indirectly calculated.
Bringing the Stars into the Lab

- Instead of both particles moving, we hold one steady (the target) and accelerate the other into it (the beam) – it's the same reaction in the center of mass frame – then we measure one or more of the reaction products.

- To measure the astrophysical rate directly, we need to accelerate the particles to the same energy that they have in the star:
  - not so easy, though: very specific (low) beam energy, and the rates are actually very small (one event in a billion or less)
  - the lower the energy (getting closer to the Gamow peak), the lower the reaction rate

- When measuring the nuclear properties (indirect), you can pick reactions which are more favorable:
  - beam energy is not so critical, and rates can be higher (one in a hundred thousand, one in a million...)
  - use a reaction which more easily populates the same nuclear levels
measuring the nuclear reactions involved in these explosive scenarios gives us insight into how they work. Now we can start to answer those questions!

(but there's still a lot we don't know)
How do we measure these nuclear reaction rates?

→ **Directly**, when possible

→ **Indirectly**, most of the time

→ **New techniques**, **new facilities**, **new equipment**...

How about an example?
The observable $^{26}$Al

1) An excess of $^{26}$Mg is found in the Allende meteorite, indicating the presence of $^{26}$Al decay...
The observable $^{26}\text{Al}$

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2) Several space-based telescopes (HEAO, COMPTEL, INTEGRAL...) observe the characteristic 1.809 MeV gamma-ray line of radioactive $^{26}\text{Al}$...

$^{26}\text{Al}$ was first radioisotope directly observed in space, by high-precision, satellite-based (expensive) instruments.
The observable $^{26}\text{Al}$

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3) COMPTEL maps the 1.809 MeV gamma across the Milky Way galaxy

4) INTEGRAL shows that the sources are near massive stars, and that the $^{26}$Al co-rotates with the galactic plane...
The observable $^{26}$Al

5) The direct observation of $^{26}$Al decay is particularly important – and useful – because of the properties of this radioactive isotope:
   - its lifetime (~700,000 years) is long enough to outlast the length of an astrophysical explosion, but much shorter than the age of typical stars or the galaxy (so it's recent)
   - it can be tracked as it moves through the interstellar medium
   - since the flux is reasonably constant, it must be actively produced in the universe in order for us to observe it!

So how is it being produced?
$^{26}\text{Al}$ in the laboratory

$^{26}\text{Al}$ is particularly interesting because it provides a direct link between the astrophysics environment and nuclear physics we can study in the laboratory.

Many reaction sequences affect the final $^{26}\text{Al}$ galactic abundance... Let's take a look at the $^{26}\text{Al}(p,\gamma)^{27}\text{Si}$ reaction, which destroys $^{26}\text{Al}$ and thus depletes the amount we could observe astronomically.

note: $^{26}\text{Al}(p,\gamma)^{27}\text{Si}$ means $^{26}\text{Al} + p \rightarrow \gamma\text{-ray} + ^{27}\text{Si}$
$^{26}\text{Al}$ in the laboratory: $^{26}\text{Al}(p,\gamma)^{27}\text{Si}$

**Direct measurement**

**Measurement of the $E_{c.m.}$ = 184 keV Resonance Strength in the $^{26}\text{Al}(p, \gamma)^{27}\text{Si}$ Reaction**

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**Observables:** just count events

- **Amount of beam necessary:** $2.5 \times 10^9$ pps
- **Amount of target necessary:** $\sim 10^{18}$ #/cm$^2$
- One level at a time

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**Diagram: DRAGON**

- **Gas Target**
- **Gamma Array**
- **Magnetic Quads**
- **Charge Slit Box**
- **Electrostatic Dipole**
- **Mass Sllit Box**
- **ICPGAC Stop**
- **MCP Start**

**Process:**
- Accelerated particle (beam)
- Target
- Recoil
- Gamma (observed by detectors)
Indirect measurement

amount of beam necessary: $5 \times 10^6$ pps
amount of target necessary: $\sim 10^{18}$ #/cm$^2$
many levels at once

measure particle transfer instead of particle capture

able to study resonances that were $10,000$ to $100,000,000$ weaker than the direct measurement!

derived reaction rate for each nuclear level in the Gamow peak

$^{26}$Al in the laboratory: $^{26}$Al(p,$\gamma$)$^{27}$Si

observables: peak energies, intensities, how intensity changes with angle, correlations between particles...
• **Direct** measurement – **great**, but very **limited**

• **Indirect** measurements – **powerful**, informative tools

• **Ongoing questions** – it's actually very **complicated**!

  - other destruction reactions?
  - creation reactions?
  - competing reactions?
  - higher energy resonances?
  - decay modes?
  - lifetimes?
  - astrophysical environment?

...our knowledge of $^{26}\text{Al}$ in the universe is still very limited!

• **Other isotopes** of interest to astrophysics but which are even more difficult to study?

  **How do we proceed?**
Pushing the Boundaries of Nuclear Astrophysics

- New techniques
- New facilities
- New equipment
using FR-ADWA vs standard DWBA results in a much better match between data sets in $^{10}\text{Be}(d,p)^{11}\text{Be}$

measuring at several energies and allowing the fit parameters to vary gives much more accurate SF and ANC values in $^{48}\text{Ca}(d,p)^{49}\text{Ca}$

New Techniques

some nuclear parameters require theoretical models to be extracted from data
New Detector Systems

ORRUBA, SuperORRUBA, HAGRiD, VANDLE, GODDESS, SECAR...

- Lots of **time**, **effort**, and **money** invested
- Several prestigious DOE Early Career **awards**
New Facilities

~$900M, completion 2022

The Facility for Rare Isotope Beams, FRIB:

→ will provide intense beams of short-lived, very exotic nuclei such as those found in exploding stars

→ will be the focus of the low energy NP community in the US, allowing us to push the boundaries of nuclear physics

→ provides multiple experimental areas for different types of studies
Low energy, exotic beams: perfect for astrophysics!

So we have all these beams being developed, but one piece is still missing...

ReA3 Hall - already providing some beams
A target is needed which is dense, highly localized, and pure

- **dense**: $\sim 10^{19}$ nuclei/cm$^2$ depending on the nuclear reaction rate to be measured ($10^{18-19} \sim$ solid)
- **localized**: target size $\sim$ beam spot size, and thin to provide good energy/angle resolution
- **pure**: scattered contaminants contribute to background, which can't be tolerated in low-stats measurements
“Million-Dollar Beams and Ten-Cent Targets”

So what do we actually use for targets?

...Development here has been largely ignored!

→ commonly using thin metal and plastic foils, implanted targets, small gas cells, which are full of contaminants and easily degrade

→ these types of targets won't work for everything... we're not leveraging the major developments being made in other aspects of nuclear astro
A Solution? Gas Jet Targets

Create a jet of light gas (helium or hydrogen) – with the correct engineering, a target that is
dense, pure, homogeneous, and localized
can be produced... state-of-the-art targetry!

We have designed, built and tested the Jet Experiments in Nuclear Structure and Astrophysics (JENSA) gas jet target, a ~$2M, 3 year, multi-institutional project
Commissioning done at ORNL, system now on dedicated beamline (one of only three) at ReA3!

Basic Components

RESERVOIR → JET
JET → PUMPING
PUMPING → GAS CLEANING
GAS CLEANING → COMPRESSOR
COMPRESSOR → RESERVOIR
large target chamber to accommodate next-gen detector systems

pumping stages (turbos) with restrictive apertures provide vacuum

turbo on central chamber backed by a roots blower

series of large roots blowers for inner and outer receiver move large volumes of gas

roots blower stages backed by multistage roots (msr) pumps to handle gas flow

custom compressor to return gas to high reservoir pressure

scroll pump for evacuating system during startup/shutdown

control and monitoring systems
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control and monitoring systems
...and most important, the Jet

- utilizes two different laval (convergent-divergent) nozzles (0.8mm and 1.1mm “neck”) - like the inside of a jet engine!
- receivers set below the nozzle catch the expanding gas (various sizes to match jet)
- 14mm “free jet” region (adjustable)

notice all the space for detectors
**Test Setup**

*Confirming our density achievement*

**energy loss** of alpha particles through the gas is linearly related to the **gas areal density** (number of target particles per area “slice”)
He Jet, 1.1mm-Neck Nozzle

Each datum is one pixel of the detector → we can “map” the density distribution of the jet

$10^{19}$ He/cm$^2$

5mm FWHM

WORLD RECORD DENSITY!

ORNL Significant Achievement Award
\(^{120}\text{Sn} + \text{nat} \text{N}_2\) Elastic Scattering at 40MeV

no target ladder shadowing!
major improvement for indirect studies
So... what next?

The best way to demonstrate the full capability of the JENSA gas jet target is to give another astrophysics example.
Another Astrophysical Observable? $^{18}$F

$^{18}$F(p,α)$^{15}$O
$^{17}$F(p,γ)$^{18}$Ne(β$^{+}$)$^{18}$F

isotope-specific signature →
$^{18}$F in novae: $^{18}$F(p,α)$^{15}$O

- Direct measurements have been made at some energies, but the rate is so small that we weren't able to determine everything (recall we need high beam intensity and long experiments...)

- The biggest gap in our knowledge of this explosive stellar reaction rate is the behavior at energies below the reaction threshold – particularly, the parameters of a single nuclear level!

Results from HRIBF $^{18}$F(d,n)$^{19}$Ne

<table>
<thead>
<tr>
<th>$E_r$ (keV)</th>
<th>$I^+$</th>
<th>$\Gamma_r$ (keV)</th>
<th>$\Gamma_0$ (keV)</th>
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<td>7.19 x 10^{-8}</td>
<td>11.624 or 0.44</td>
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<td>8</td>
<td>3/2</td>
<td>7.19 x 10^{-10}</td>
<td>0.5</td>
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<td>4.0</td>
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<tr>
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<td>1.2 x 10^{-5}</td>
<td>1.2</td>
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<td>2.7</td>
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<td>1122</td>
<td>5/2-</td>
<td>10.0</td>
<td>21.0</td>
</tr>
</tbody>
</table>

(Bardayan 2001,2002 And Beer 2011)
Indirectly: $^{20}\text{Ne}(p,d)^{19}\text{Ne}$

- Problem is interference – the nuclear spin of the level in question manifests in the quantum wavefunction of that state, and this wave can interfere constructively or destructively with other resonances.

- All we need to know is the spin of that state, which we can determine by studying the angular momentum transferred to/from that level – this is a perfect opportunity for an indirect study!

- With a proton beam and a $^{20}\text{Ne}$ target, use the (p,d) transfer reaction to populate levels in $^{19}\text{Ne}$ – the same levels in this compound nucleus as during $^{18}\text{F}(p,\alpha)^{15}\text{O}$:

$$^{18}\text{F} + p \rightarrow ^{19}\text{Ne}^* \rightarrow ^{15}\text{O} + \alpha$$
This reaction was studied before we had JENSA, using a carbon foil implanted with neon atoms...
$^{20}\text{Ne}(p,d)^{19}\text{Ne}$

Identify peaks by their energy ("kinematics")

A fair amount of unwanted background down here...
$^{20}\text{Ne}(p,d)^{19}\text{Ne}$

Counts vs. Ejectile Energy (MeV)
$^{20}\text{Ne}(p,d)^{19}\text{Ne}$

![Graph showing neutron transfer reactions]

- $^{12}\text{C}(p,d)^{11}\text{C}$
- $^{16}\text{O}(p,d)^{15}\text{O}$
- $^{16}\text{O}(p,t)^{14}\text{O}$

Counts vs. Ejectile Energy (MeV)
$^{20}\text{Ne}(p,d)^{19}\text{Ne}$

There must be a better way...
try again with JENSA
$^{20}\text{Ne}(p,d)^{19}\text{Ne}$
The distribution in **counts vs angle** of the deuterons (d) from the reaction tell us about the **transferred angular momentum** - that tells us the **spin of the nuclear level** they came from...
Demonstrating the Power of JENSA

Matching that distribution to theoretical curves confirms it: the properties of this important nuclear level have been indirectly measured via $^{20}\text{Ne}(p,d)$ using JENSA, filling the remaining gap in our understanding of $^{18}\text{F}(p,\alpha)^{15}\text{O}$.
On to the Future

• There are a multitude of opportunities for important, astrophysics-motivated, indirect measurements of nuclear properties just like the examples I've shown!

• Many indirect (and also some direct) measurements are possible, using JENSA gas jets of $^1$H, $^2$H (D), $^3$He, and $^4$He – and someone needs to lead the effort

• Many beams from ReA3/FRIB are those of interest to astrophysics, most of which have not been available before now (others are available at much higher intensities than ever before)

• Improvements to reaction formalism/theory mean we can be more confident of our derived reaction rates
The Observables: $^{18}\text{F}$ and $^{26}\text{Al}$

The expected beam intensities for ReA3/FRIB
The Observables: $^{18}\text{F}$ and $^{26}\text{Al}$

The expected beam intensities for ReA3/FRIB
The Observables: $^{18}\text{F}$ and $^{26}\text{Al}$

The expected beam intensities for ReA3/FRIB

Lots of beam for indirect studies with JENSA
One Intriguing New Possibility: $^{56}$Ni, Another Astrophysics Observable

LETTER

doi:10.1038/nature13672

Cobalt–$^{56}$Co γ-ray emission lines from the type Ia supernova 2014J

→ observation of $^{56}$Co gamma rays indicates the presence of $^{56}$Ni: $^{56}$Ni → $^{56}$Co → $^{56}$Fe

→ $^{56}$Ni can be a “thermometer” which gives detail of supernova explosion mechanism

→ $^{56}$Ni is also a “waiting point” nucleus in the rp-process: a bottleneck in element synthesis

Beam intensities sufficient for indirect studies are expected!

Map of the $^{56}$Co gamma rays around SN2014J: “The line fluxes suggest that about 0.6±0.1 solar masses of radioactive $^{56}$Ni were synthesized during the explosion.”
One Intriguing New Possibility: \( ^{56}\text{Ni} \), Another Astrophysics Observable

Cobalt-\( ^{56}\text{Ni} \) is also a “waiting point” nucleus in the rp-process: a bottleneck in element synthesis.

Beam intensities sufficient for indirect studies are expected!

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**Letter of Intent: The Next Generation of JENSA-Driven Measurements**

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**I. INTRODUCTION**

The Jet Experiments in Nuclear Structure and Astrophysics (JENSA) Collaboration gas jet target is ready for its first experimental campaign in the ReA3 hall. The details of the JENSA gas jet target may be found elsewhere [1–5]. Commissioning tests have indicated that JENSA can reliably provide areal densities up to \( \sim 1 \times 10^{19} \text{ atoms/cm}^2 \) helium [5]. The anticipated densities for hydrogen, deuterium, and \( ^{3}\text{He} \) are similar [4]. For each of these gases, the use of a dense, localized, and pure gas jet target affords tremendous advantages over traditional targets [5].

This Letter of Intent describes measurements of the following reactions: \( ^{26}\text{Si}(\alpha,p) \), \( ^{30}\text{S}(\alpha,p) \), \( ^{30}\text{P}(^{3}\text{He},d) \), \( ^{56}\text{Ni}(^{3}\text{He},d) \), \( ^{5}\text{Ni}(\alpha,p) \), and \( ^{56}\text{Ni}(d,p) \). The Collaboration therefore requests that development of these beams be prioritized by the facility.
The Take-Home Message

Combining the powerful capabilities of the JENSA gas jet target (of which I am technical lead and PI), exotic beams of astrophysical nuclei from FRIB, and the latest nuclear reaction theory, we can probe more deeply and thoroughly into the stars than was previously possible.
Thanks

The ORNL Physics Division
(esp. David Dean, Michael Smith, Steven Pain)
The JENSA Collaboration
(esp. Dan Bardayan, Antonios Kontos, Allison Sachs, Paul Thompson)
The RIBENS Collaboration
(esp. Patrick O'Malley)
Extras
From the Stars to the Lab

proton  neutron  gamma

Inside a star

stellar temperature $kT$

release of energy
From the Stars to the Lab

proton  neutron  gamma

In the lab

release of energy

detectors
From the Stars to the Lab

proton  neutron  gamma

In the lab, “inverse kinematics”

release of energy

detectors
From the Stars to the Lab

proton  neutron  gamma

direct

indirect

Gamow peak

\( a+b \)

\( Q \)

\( \text{Ex} \)

\( \text{Er} \)

Gamow peak

\( c+d \)

\( a+b \)

\( Q \)

\( \text{Ex} \)