# Commotion and Catamarans

Jeff Smith

Faculty Sponsors: Robert H. Hoar and Robert J. Ragan,
Department of Mathematics

## ABSTRACT

Many problems in our world, can be modeled using mathematics. These models can then be used to make approximations of what might happen given certain outside forces and other circumstances. The research dealt with modeling a catamaran (a type of water-craft) and its reaction to certain forces in water. We began by modeling a one-mass craft and set up some basic differential equations to model certain force effects on the mass. We then expanded to a more realistic catamaran (which is a 3 mass system) and modeled this with two sets of differential equations. After studying the geometry, we were able to correctly model certain aspects of the catamaran. We then used mathematics software to study the effects of certain forces. We found that certain factors must be taken into account in order to keep the boat stable.

## INTRODUCTION

The research I conducted dealt with the actions of a watercraft, a catamaran, given certain outside influences. The first order of business was to collect all influential forces and how they would react on the catamaran. After these forces were determined we then needed to put together, or couple these forces to come up with a proposed mathematical model which would predict what the catamaran boat would do given certain initial influences. Mathematical modeling played a large role in our research project. Putting real world events into a mathematical formula of equations took much time and scrutiny. Every force eventually must be taken into account. In order to get a grasp on the model at all, though, we first had to simplify the model into something more workable and mathematically familiar. This in turn will eventually lead a more realistic model.

Our expected outcome of this project was to create a realistic mathematical model that would give insight into what would be the catamaran's reaction to certain events. We expected to boat to bob up and down as well as have some angular displacement from equilibrium given the water's effect. We wanted to introduce as many outside forces as possible into the model and still make accurate predictions about the catamaran.

## MATERIAL AND METHOD

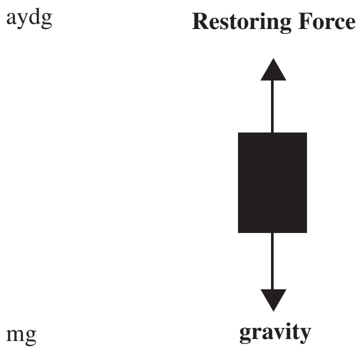
In conducting our research, Dr. Hoar, Dr. Ragan, and myself took into account many outside forces. These included wind, water, gravity, waves or rough water, outside pushes, velocity from sails and the wind, human interactions from sailing, and last but not least, friction. These forces could move the boat vertically, horizontally, and rotationally. However it was unrealistic to introduce all these variables at once so we started with a small simple model of a cork. This is more

workable. We considered a single mass sitting in the water.

Also, we only considered forces in there vertical direction. These included the restoring force or buoyancy, and gravity acting downward. This is now a moving body problem. This deals with Newton's second law. Newton's second law is F=mA, where F is the sum of the forces acting on the object, A is the acceleration due to gravity, and M is the mass of the cork. We assumed there was no friction and all forces acted upon the center of the mass. It should be noted that A is also the second derivative of displacement with respect to time, which is what we are trying to predict. Our two forces are mass multiplied by gravity in the downward direction and the area of the cork in the water multiplied by the density of the water multiplied by gravity in the upward direction. Our model now looks like:

$$F=mA=my''(t)=-mg+aydg$$

<where a is the horizontal cross-sectional area, y is the depth of the bottom of the cork, and d is the density of water.




This can now be written as a second order differential equation in terms of y(t). A second order differential equation is just an equation involving derivatives and second derivatives.

$$y''(t)= (aydg)/m-g$$

By taking y(t) to equal the initial y displacement and the change in y(t0 we can do a change of variables and our new equation is easier in terms of how solvable it is.

$$y''= -(adg/m)y$$

This is because of the fact that y(0) = (m/ad) because of balance in water. This equation is easier to solve because it is in the form of a constant (adg/m) multiplied by the change in y(t).

Now that we had conceived this second order differential equation we worked towards solving the equation. Using basic differential techniques, our second order equation becomes a system of two equations which are first order differential equations. Our two equations in first order are:

$$y1= y'$$
$$y2= y1'= y'' = -(adg/m)y$$

Both equations involve nothing higher than first derivatives. To solve this we again used basic differential equation techniques. These techniques are summarized in the following:

Note: Eigenvalue: ev, (adg/m)=q

Ev^2 + (-q)= 0  so ev = +or- sqrt(-q)

Matrix subtraction (A-ev1):

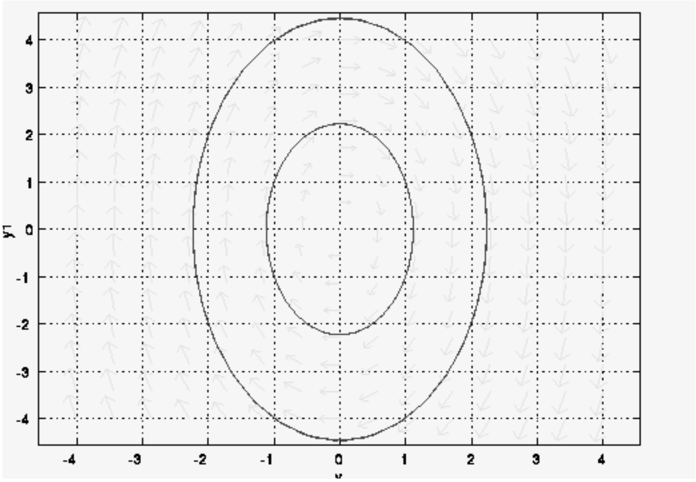
eigenvector

y1 | 0 1 | - | sqrt(-q) 0 | = | sqrt(-q) 1 | 0 | 1 |
y2 | -q 0| | 0 sqrt(-q)| | -q sqrt(-q)| 0 |sqrt(-q)|

e^ev * eigenvector = y(t)

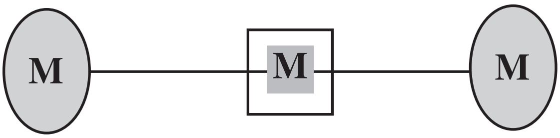
y(t)={cos[sqrt(q)t]-i*sin[sqrt(q)t]}*eigenvector

y(t) is a linear combination of these real and imaginary values. This linear combination of sines and cosines looks like the following in a phase plane:




In this picture, displacement is in the vertical direction and velocity is in the horizontal direction. Basically the picture says the cork is moving up and down at varying speeds given the initial one unit push. We now know what is happening with the cork in the vertical direction.

Next, we worked on expanding the cork model into a more model of the catamaran. For the most part we used the same assumptions, but now we will introduce another displacement, angular displacement. To do this we will look at two hull model.




The forces in the vertical direction are still the same and act on the center of the mass, M. The new mass if the model is now (M+2m). Our new second order differential equation is:

Y"= -(ad/(M+2m))y

Much like our cork model equation. To find the second order differential equation for angular displacement, we again look at Newton's second law, but now for angular motion, or torque (T).

For our two hull model:

T= I Θ" = F1r1 - F2r2

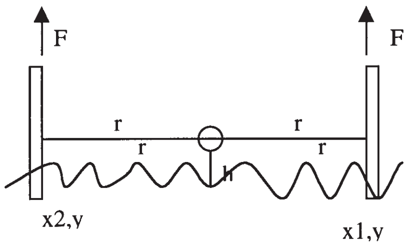
F=uadg

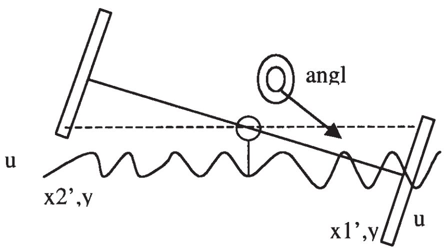
-Note:u=displacement of craft underwater.

I=2mr^2

I=moment of inertia

This fits our model:




This must keep correct geometry throughout, so we list our geometry:

Necessary variables

(x1,y1)=(r,-L)

(x2,y2)=(-r,-L)

xa=xcos Θ -ysin Θ

ya=xsin Θ +ycos Θ

u1,u2: amount of hull underwater

uya = -ya - h

vertical direction

u1= uy1/cos Θ

We are now ready to introduce the new motion into our mathematical model. We now have our second order differential equation for angular displacement after much trigonometry and algebra:

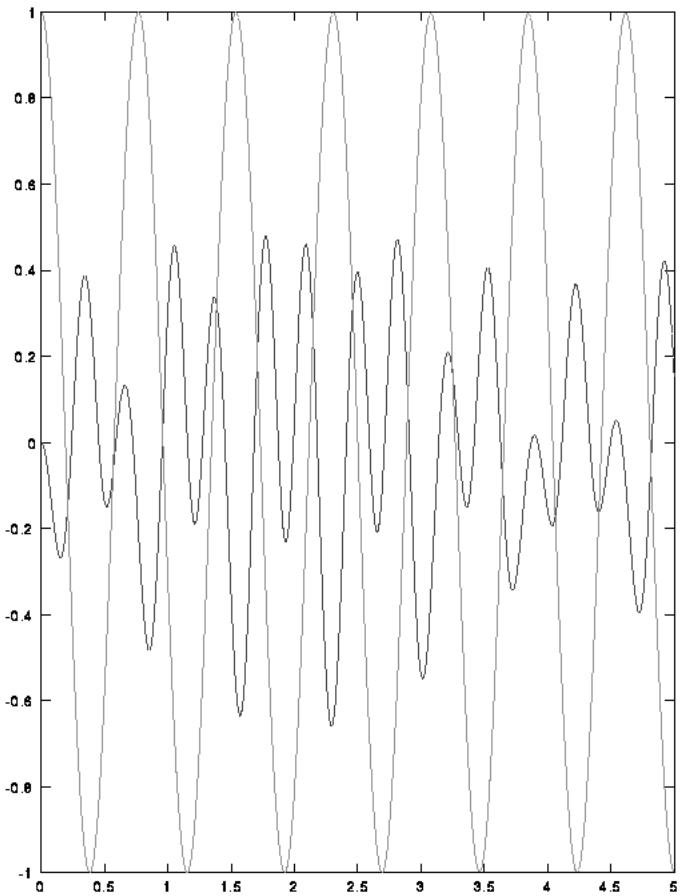
Θ"= - (adg/m)[1-(1/2r^2(L^2-h^2)]Θ

We then coupled this with the other second order differential equation. To solve this we now have system of four coupled first order differential equations.

## RESULTS

-rhs(1)=y(3)
-rhs(2)=y(4)
-rhs(3)=(-2*A/(M+2m))*y(1)
-rhs(4)=((A/(2*m*(r^2)))*((-(y(1)))*((2*r)-(2*L*c)+(r*b))+a*((-(2*(L^2))-...
(2*(r^2))+(L*r*c)-((r^2)*b)))))

Using Matlab we were able to obtain the numerical results in the following plot. In these Matlab results we used a second and third order Runge-Kutta Method to couple the four equations. We then used certain initial pushes of one unit on the right side to start the catamaran moving. The horizontal axis is time and the vertical axis is displacement. Basically the boat is moving around at different speeds up and down and angularity tipping throughout. The curve which moves between -1 and 1 is the vertical change in displacement. The other curve shows the angular displacement in radians.



## CONCLUSION

Our results are what we expected, but they are still not perfect. They do support our hypothesis of movement in many aspects. Our model incorporates many forces and has become more and more realistic. Integrating the geometry and algebra and physics along with the differential equations was very complicated and took much scrutiny and research. The research was successful and concise, but can always be improved upon. Further research could be conducted in this area by adding in even more precise details or even adding the effects of a human sailor on the catamaran. Our theories and explanations can always be built upon for even more clarity.

## LIMITATIONS

It is quite difficult in coupling to make sure the right forcing terms that affect angular motion and vertical motion are in each equation. Furthermore, linearization was used and could have slightly inhibited the results. There is also the limitation of not having used friction and other additional natural forces.

## REFERENCES

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