NMR Spin-Echo Investigations of Magnetized Quantum Fluids: Effect of Cavity Walls on Spin-Wave Instabilities

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ABSTRACT

We report computer simulations to determine the effects of cavity walls on spin waves in spin-polarized quantum fluids. We have written a Java program to perform a finite-difference calculation that simulates the propagation of spin waves in a magnetized sample of liquid.^{3}He during a pulsed NMR spin-echo experiment. The NMR signal from such experiments is used to study spin transport and spin interaction in quantum fluids. Our simulations show how the ends of the NMR cavity distort and destabilize the spin waves, and how this in turn distorts the NMR signal. Our simulations also reveal the effect of such experimental parameters as temperature, polarization, field gradient strength, and pulse intervals on the growth of the instabilities. Such simulations can be used to develop strategies to minimize the growth of instabilities, as well as to assess the validity of previously published experimental data.

INTRODUCTION

The unusual features of spin dynamics in spin-polarized Fermi fluids have been intensively studied since the pioneering work of Leggett and Rice (LR) on spin diffusion in normal liquid ^{3}He[1]. At sufficiently low polarizations the spin dynamics is well described within the framework of the Landau theory of Fermi liquids [1] [2], and is characterized by ordinary spin longitudinal spin diffusion, and damped transverse spin waves with quality factor μM. In 1985, Meyerovich suggested that in high magnetic fields spin transport should be anisotropic because of a zero-temperature attenuation of transverse spin currents at finite polarization[3-5]. More recently, quantitative calculations have been reported of transverse and longitudinal spin diffusion coefficients, $D_\perp$ and $D_\parallel$, for very dilute ($x_3 << 1\%$) ^{3}He-^{4}He solutions at finite polarizations [6]. The longitudinal spin diffusion coefficient $D_\parallel$ has the usual behavior of Fermi-liquid transport coefficients and diverges like $1/T^2$ as $T \rightarrow 0$. On the other hand, $D_\perp$ remains finite for finite polarizations even at T=0 because of the phase space volume available for scattering between the spin-up and spin-down Fermi surfaces. Recently, the reduction of the spin diffusion coefficient has been observed, but only in pure ^{3}He[7] and 3.8% ^{3}He-^{4} He solutions [8], for which no quantitative theory exists. On the other hand, the existence of zero-temperature attenuation in liquids has been questioned be Fomin [9], who has derived an alternate, undamped, spin-wave spectrum at zero temperature. Also, tentative experimental evidence for undamped spin waves at ultra low temperatures has been recently reported [10]. To settle the controversy, precise experimental tests are needed of the quantitative theoretical predictions for very dilute ^{3}He-^{4}He solutions. Since the Fermi temperature is $\propto x_3^{2/3}$ this requires very low temperatures. Spin-echo experiments are currently being prepared

[11] at the High B/T facility at the National High Magnetic Field Lab (NHMFL) for temperatures down to milliKelvin and B=17 Tesla for concentrations below 1%. At such low temperatures it is anticipated that diffusion will be highly anisotropic, $D_{\perp} < 0.5\ D_{\parallel}$. However, at these temperatures the spin-rotation parameter will be very large $\mu M > 100$, and for large μM spin currents are known to be unstable against long wavelength perturbations. The so-called Castaing instability [12] is known to lead to the spontaneous formation of domain wall [13- 14] and is thought to be the source of anomalous long-lived signals in longitudinal spin diffusion experiments [15]. Naturally, the question arises of whether the effect of instabilities can mimic or mask the effects of anisotropy. In order for the NHMFL data to be unambiguously interpreted, the effects of spin-wave instabilities in spin-echo experiments must be understood. The results of a few spin-echo experiments have been reported where departures from theory have been observed and tentatively associated with instability [7,8]. Recently, Ragan has analyzed the effect of the Castaing instability in simple spin-echo experiments and has shown that instability can interfere with observation of zero- temperature attenuation [16]. The analysis reveals the polarization and temperature where the instability should set in, and also reveals the importance of tip angle and field inhomogeneities. The study consists of a linear stability analysis, to investigate the onset of stability, and computer simulations to study the development of distortions of the magnetization and NMR signals. However, the results are of a preliminary nature since the effects of the NMR cavity walls were not considered. Earlier Ragan had analyzed the effects of boundary, and determined that distortions of the magnetization will result[17]. The effect of the boundaries should be important if

$$\mu M\gamma GL^3/D_{\perp} >> \pi^4/4 \qquad \text{Eq. 1}$$

where γ is the gyromagnetic ratio, G is the magnetic field gradient, and L is the NMR cavity length. However, this analysis did not take instabilities into account. The purpose of this project is to write a Java program, so that the Eq. 1 can be tested when instabilities are included.

Since the Castaing instability is present in Leggett's original hydrodunamic-type equations, we shall avoid unnecessary complication by neglecting anisotropy and polarization dependence in the spin diffusion coefficients. n this case, the spin current is given by the quasi-steady value

$$J = -\frac{D}{1+\mu^2 M^2}\left(\partial M+\mu M \times \partial M+\frac{1}{2}\mu^2\partial(M^2)M\right) \qquad \text{Eq. 2}$$

Where μM is the spin rotation parameter. Equation (1) determines the dynamics of the magnetization density through the continuity equation,

$$\partial_t M+\gamma B_{loc}=-\partial J \qquad \text{Eq. 3}$$

B_{loc} is the local microscopic field experienced by the spins. For simplicity, we shall assume that the sample volume is a thin tube with its long axis oriented along the z-direction, so that

$$B_{loc}=B_0+Gz\hat{z}-\mu_0(M_z\hat{z}+M/3) \qquad \text{Eq. 4}$$

Where B_0 is the polarizing field, G is the imposed field gradient and $\mu_0 M_z\hat{z}$is the "local" demagnetizing field.

In a θ - Δt - π experiment the initially uniform equilibrium magnetization is tipped at t=0 with an RF pulse about, say, the x-axis by an angle θ. For $t<\Delta t$ the transverse magnetization twists via Larmor precession into a helix of wavenumber $k_0(t)=\gamma Gt$. As the helix evolves, the magnetude of the magnetization remains spatially uniform and the z-magnetization is usually assumed to be uniform and constant, $M_z=M_0\cos\theta$. At t= Δt a π "time reversal" pulse is applied whose effect is mz Æ -mz and m+→ m-. After the π pulse the helix unwinds according to $k_0= \gamma G(2\Delta t -t)$ until a spin echo occurs at $t=2\Delta t$ with a uniform megnetization density. The spin diffusion coefficient D and spin interaction parameter μM can be determined from the height and direction of the spin-echo which is detected with an RF antenna (see Figure 1).

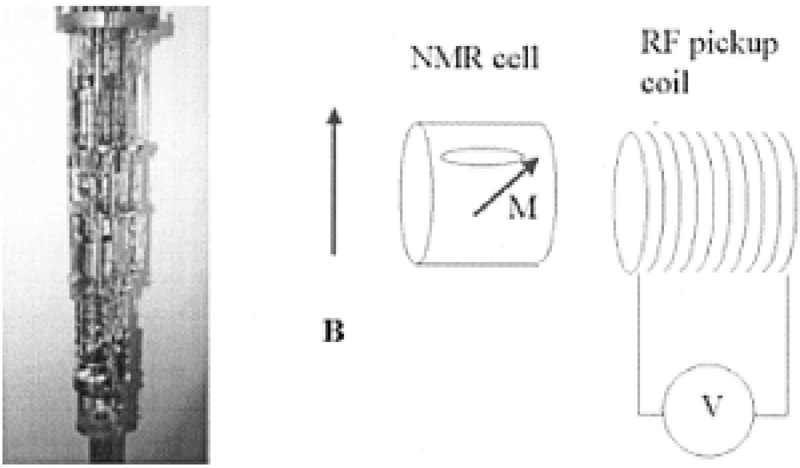

Figure 1. Dilution Refrigerator with Nuclear Demagnetization Cooling Stage and schematics of NMR setup

METHODS

The equations of motion were integrated with a real-space finite-difference scheme using a fourth-order Runge-Kutta method for the time steps. To insure numerical stability it usually proved sufficient to use the von Neuman condition for ordinary diffusion $(\Delta z)^2/\Delta t\geq 2$.

A Java applet was written to do the simulations; this applet lets the user set the experimental parameters N_{turn}, D*, μM, and θ. Where $N_{turn}=L\gamma G\Delta t$ represents the number of turns of the helix in the cavity, $D^*=D\gamma^2G^2\Delta t^3$ is a dimentionless parameter measuring the pulse time, μM is the spin interaction parameter and θ is the tip angle. The program generates two different types of output. First it creates an animation which shows the output of the RF pickup coil. Figure 2 shows what this window looks like. In figure 2 the left side window shows the magnetization inside the NMR cell, the right side window gives the experimental RF signal showing the spin echo. This applet can be used in the internet at www.uwlax.edu/faculty/ragan/xyDiff12/xyDiffTest12.html. The applet also outputs the magnetization profiles to data files.

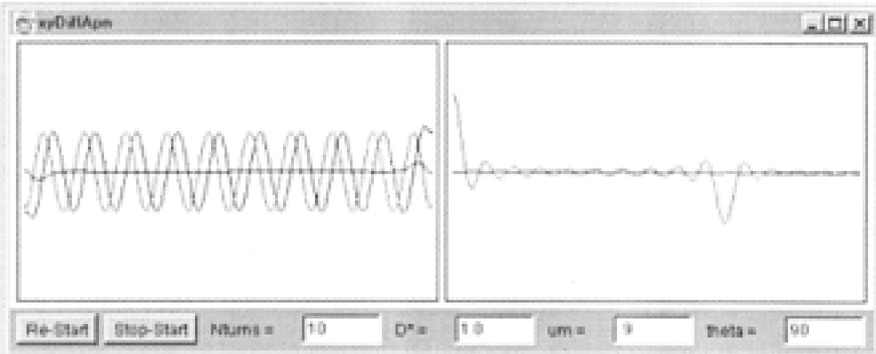

Figure 2. Applet used to simulate NMR experiments

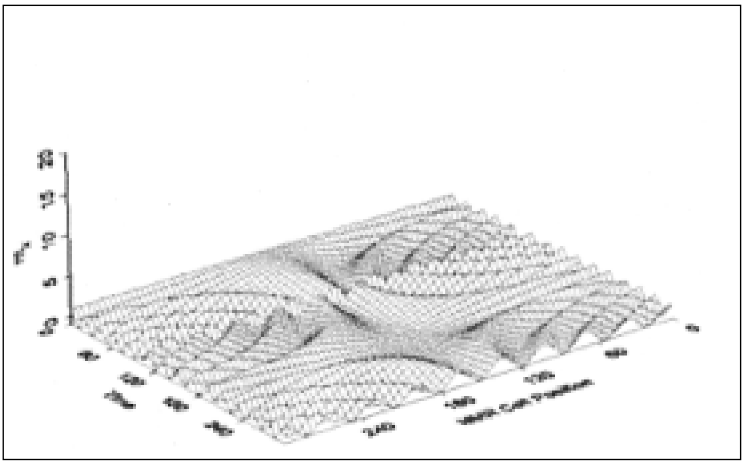

Figure 3. Propagation of spin-wave during a NMR experiment

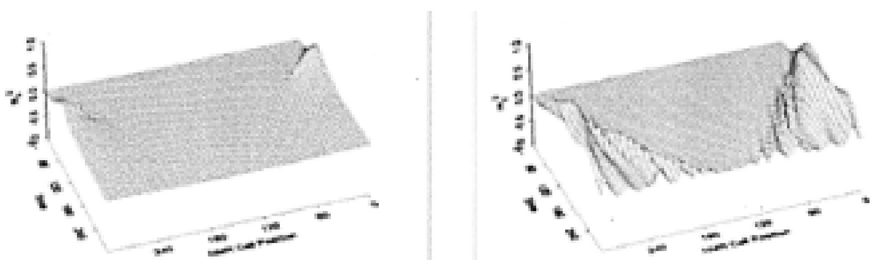

Figure 4. Distortions of spin-waves due to interactions with the walls during a spin echo experiment. The first simulation is at temperature $T=0.2T_{fermi}$ and the second is at $T=0.1T_{fermi}$ where T_{fermi} is the Fermi temperature.

RESULTS AND DISCUSSION

A quantum fluid is a whole new kind of matter, with all kinds of exotic properties, including superfluidity. The conducting electrons in a superconductor are an example of a quantum superfluid. Helium-3 is the simplest "cleanest" quantum fluid and is the easiest to study. Spin-polarized He-3 is studied in order to understand the interactions of atoms in a magnetized quantum fluid.

Our simulations show the ends of the NMR cavity distort and destabilize the spin waves, and how this in turn distorts the NMR signal. Our simulations also reveal the effect of such experimental parameters as temperature, polarization, field gradient strength, and pulse intervals on the growth of the instabilities. Such simulations can be used to develop strategies to minimize the growth of instabilities, as well as to assess the validity of previously published experimental data.

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