

Improving the Durability of Duration

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ABSTRACT

Duration is one of the most important tools for measuring the risk changing interest rates pose to an investment. Like many theoretical tools, however, the assumptions that make the tool work are often very restrictive and impractical. This research attempted to see how flawed one of the assumptions is, and what could be done to improve it.

INTRODUCTION

In 1938, an economist named Fredrick Macaulay wanted to try to find a better way to measure the maturity of bonds issued by railroad companies. The practice at the time was to say that two bonds had the same “time until maturity” if they both matured at the same time, regardless if one paid \$10 a year in interest and the other paid \$100.

Macaulay, however, noted that the prices of those two bonds would behave very differently. He wasn’t satisfied that time until maturity was the right way to classify what he termed the “longness” of the bond. He felt that the intermediate cash flows (typically coupon payments) needed to be considered as well. In his influential, if not oddly titled, book *The Movement of Interest Rates, Bonds, Yields, and Stock Prices in the United States Since 1865*, he proposed a measure, which he dubbed “duration”, that he felt provided a more accurate picture about the maturity of a bond than the straight time until maturity.

Macaulay’s duration measure is nothing more than a basic weighted average measure of the time to each bond payment, where the weights are the present value of the individual payment as a percentage of the total present value of all the cash flows.¹ The formula for determining duration is:

$$\text{Duration} = \frac{\sum_{t=1}^m t \cdot CF_t}{\sum_{t=1}^m \frac{CF_t}{(1+i)^t}}$$

where:

CF_t =Cash flow from the bond at time t
 m =years until maturity
 i =yield to maturity

In his book introducing duration, in order to justify why he created it, Macaulay wrote:

“It is clear that number of years to maturity is a most inadequate measure of duration. We must remember that the maturity of a loan is the date of the last and final payment only. It tells us nothing about the sizes of any other payments or the dates on which they are made. It is clearly one of the factors determining duration...Duration is a reality of which maturity is only one factor.”

To see an example of how bonds that may have wildly different characteristics have the same duration, and therefore why time to maturity is a poor measure of bonds, please refer to the appendix of this article.

However, there is a major assumption in Macaulay’s duration model that has plagued it since it first started gaining widespread use in the 1970s. For his model, Macaulay assumed that interest rates will be the same across all maturities, which is evidenced by the fact that the same discount rate is used to find the present value of each cash flow, from the first until the last. He also assumed that when interest rates change, they will change by the same amount across all maturities. As you can see from the following yield curves, interest rates are rarely flat and it is even more rare that there are parallel shifts in the curve. In general, interest rates change much more drastically the shorter the maturity.

It is this broad generalization that this research project hopes to address. By looking at how interest rates have moved in the recent past and analyzing that data, it is hoped that a volatility-weighted duration can be developed that will remove this generalization

METHOD

In order to create a volatility-weighted measure, the first step was to go to the US Treasury Department website and find the yield to maturity on various US treasury issuesⁱⁱ, since it is these issues that are most often used as a proxy for general changes in the interest rates of the market. The government issues that were analyzed were the 90-day, 180-day and the one, two, three, five, seven, ten, twenty and thirty year issues. For each issue, monthly data was collected from the most recent data available and then going back 5 years. This meant that each issue had 60 data points to analyze.

Once the data was collected, a simple standard deviation of the rates for each issue was computed to see the relative price volatility across different maturities.

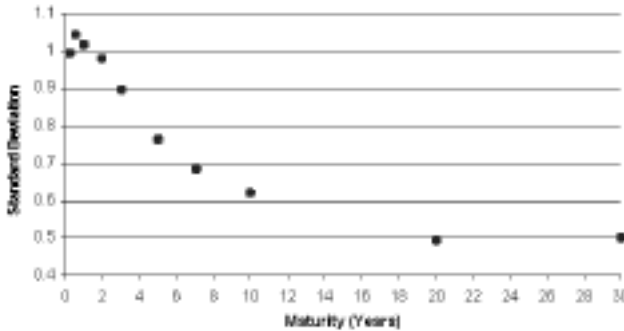
After finding the standard deviations, a function was desired that could linearly model the data. Using some linear regression tools, a linear equation of order four was found, which would allow us to approximate the volatility for time periods without data (for instance, how volatile would a six-year issue be?)

RESULTS

The various issues and the standard deviations of their rates are listed in the following chart:

| | | | | | |
|---------|--------|-------|--------|--------|--------|
| 90 Day | 0.9942 | 3 Yr. | 0.8987 | 10 Yr. | 0.6204 |
| 180 Day | 1.0453 | 5 Yr. | 0.7632 | 20 Yr. | 0.4919 |
| 1 Yr. | 1.0166 | 7 Yr. | 0.6849 | 30 Yr. | 0.9942 |
| 2 Yr. | 0.9795 | | | | |

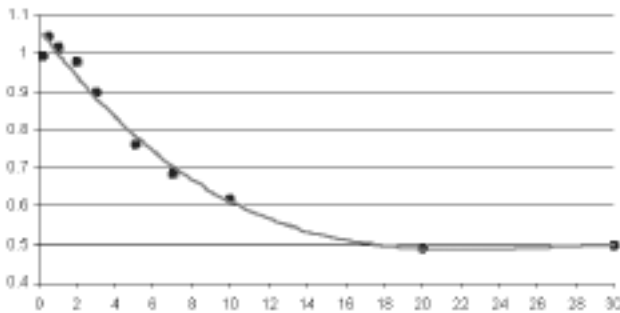
Graphically, the data plots as:



Using Statistical Linear Regression calculator on the University of Wisconsin-La Crosse website¹¹¹, the third degree function that best fits this set of data is:

$$Y = (-3.2512 \times 10^{-5})x^3 + 0.0026235x^2 - 0.068281x + 1.0653$$

which plots against the data as:



The statistical measure, R-Squared, tells how well a regression line “fits” the data it was derived from. For this regression line, the R-Squared value is 98.5%, meaning that 98.5% of the variability in the data is measured by the equation that we have come up with. Statistically speaking, this is a very good fit.

CONCLUSIONS

As evidenced graphically, the assumption inherent in the traditional duration model that the interest rate curve will make parallel shifts across all maturities is false. Clearly when shifts occur in the interest rate curve, the shifts are more extreme in the shorter maturities than in the longer maturities.

In order to improve the current duration model by incorporating this knowledge, we could weight each cash flow that the investment produces by a measure of volatility. This would tend to decrease the effects of early cash flows and increase the impact of later cash flows on duration, which would provide a more accurate measure of the interest rate sensitivity of that investment.

LIMITATIONS

There were several limitations in this project that could be improved upon in future experiments on the topic. When analyzing the standard deviation of the treasury yields, a more rigorous test could have been implemented in order to make the most recent data weigh further on the outcome than the older data. As it was analyzed, the five year old data influenced the volatility as much as more recent data. This could be alleviated by using a time series analysis that would weight the more recent data more heavily than the older data.

Another limitation of the study was the use of a simple linear regression instead of a more complex logarithmic regression that may have fit the data more accurately and captured the “spike” in the early data. However, as mentioned earlier, the R-Squared value indicates that the equation we used to model the data fit the data reasonably well. It is doubtful that a better fit would have influenced the final outcome in a meaningful way.

The last limitation of this study is that as time changes, there will be newer data that needs to be incorporated into our model, and a new line will need to be fit. This means that the equation that is found today could vary greatly from an equation found using the same methodology one year from now.

APPENDIX^{iv}

Recall that the whole reason that Frederick Macaulay created duration in the first place was that using a simple time to maturity was not sufficiently summarizing the nature of the bonds. Take for instance the following three bonds:

An 18-year bond that pays 10.5% annually and currently has a yield to maturity of 13%

A 10-year bond that pays 4.11% annually and currently has a yield to maturity of 9%

An 8-year bond that pays no interest, but is priced to yield 6%

If we didn't have duration as a tool, we might think that each of these bonds would react quite differently to changes in interest rates, with the shortest term bond being very volatile relative to the other two. It was precisely this situation that Macaulay wanted to correct with his duration model. Recall that duration is defined as:

$$\text{Duration} = \frac{\sum_{t=1}^m t \cdot CF_t}{\sum_{t=1}^m CF_t (1+i)^t}$$

where:

CF_t =Cash flow from the bond at time t

m=years until maturity

i=yield to maturity

If you were to find the duration of the three bonds, you would discover that each has a duration of approximately 8 years, which means that each of the three bonds are practically identical in how they will react to changes in interest rates. The computation of the duration for the 10 year bond, assuming that it is worth \$1000 at maturity, is shown below as an example:

| 1 | 2 | 3 | 4 | 5 |
|-----|-------------|-----------|-----------------|-----------------|
| t | CF_t | $(1+i)^t$ | $(1^*2)/3$ | $2/3$ |
| 1 | 41.1 | 1.09 | 37.70642 | 37.70642 |
| 2 | 41.1 | 1.1881 | 69.1861 | 34.59305 |
| 3 | 41.1 | 1.295029 | 95.21022 | 31.73674 |
| 4 | 41.1 | 1.411582 | 116.4651 | 29.11628 |
| 5 | 41.1 | 1.538624 | 133.5609 | 26.71218 |
| 6 | 41.1 | 1.6771 | 147.0395 | 24.50659 |
| 7 | 41.1 | 1.828039 | 157.3818 | 22.48311 |
| 8 | 41.1 | 1.992563 | 165.0136 | 20.6267 |
| 9 | 41.1 | 2.171893 | 170.3122 | 18.92358 |
| 10 | 1041.1 | 2.367364 | 4397.719 | 439.7719 |
| | Sum: | | 5489.595 | 686.1765 |

Dividing 686.1765 into 5489.595 shows that the duration for this bond is eight years.

ⁱ *Duration: Its Development and Use in Bond Portfolio Management*, G.O Bierwag

ⁱⁱ <http://www.treas.gov/domfin/hist1yc.htm>

ⁱⁱⁱ http://www.compute.uwlax.edu/stats/multi_stats.php

^{iv} Adapted from *Fundamental of Investing, Seventh Ed.*, p.366