

# Volatility of Eurodollar Futures and its Impact on Hedging

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## ABSTRACT

There are many inherent risks in the world of finance. Several techniques may be employed to reduce the risk exposure. This paper will examine the use of Eurodollar futures contracts in hedging and risk reduction. A difficulty using futures to hedge risk arises due to the volatility of the futures contracts coupled with the volatility of the position that is to be hedged. The hedge ratio assumes constant volatility of both the underlying position and the futures contract. Previous research shows the volatility of the underlying position may vary over time. This research will measure the volatility of Eurodollar futures contracts over time and the impact on the hedge ratio.

## INTRODUCTION

Hedging risk is essential to anyone who is faced with an open financial position. The fear is that this position could appear to be favorable at present, but become just the opposite by the time the position matures or is closed. This fact is very important to anybody needing to deal with risk exposure. An open position could arise if a domestic company knows that they will receive \$5 million Swiss Francs in six months. The company is faced with the possibility that the Swiss Franc will depreciate in relationship to the US Dollar. To protect against this occurrence, the corporation could enter into an equal and opposite position in the futures market. In other words, they might choose to take a short position in the Swiss Franc futures market. If the Swiss Franc did depreciate in relationship to the US Dollar, the futures position would profit offsetting the loss in the underlying position. If the Swiss Franc appreciated, gains in the underlying position would be offset by the loss on the futures position. Yes, if the Franc appreciated and the company did not hedge, they would have realized a profit. However, the goal is not profiting from these unavoidable fluctuations, but to provide risk protection when the price moves unfavorably. In this case, the short position in the hedge would have offset the gain in the Swiss Franc spot price, but more importantly prevents losses if the spot rate swung in the opposite direction.

This paper is a continuation of an undergraduate research project, Stuart (2002). Stuart (2002) looked at the volatility of bond prices and created a volatility-weighted measure of duration. The results suggested that as maturity approaches, the interest rate volatility of bonds increases. This is in contrast to a common assumption that interest rates experience parallel shifts across all maturities. In truth, there is volatility and the volatility itself undergoes changes over time.

This study is intended to prove a similar truth about Eurodollar futures contracts. When using them as a hedging device, one must be very concerned with the volatility of these contracts. As volatility increases, more inherent risk is assumed. The results of this study coupled with that of Stuart (2002) will help to find the ideal maturity of futures contracts that is necessary to employ and maintain a balanced hedge ratio. Hull (2000) provides the equation for the hedge ratio,

$$h = \rho (\sigma_s / \sigma_f) \quad (1)$$

where:

h = Hedge ratio (the number of futures necessary to balance or hedge our underlying position)  
 $\rho$  = Correlation coefficient between the spot and futures interest rate  
 $\sigma_s$  = Standard deviation of the spot rate  
 $\sigma_f$  = Standard deviation of the futures rate

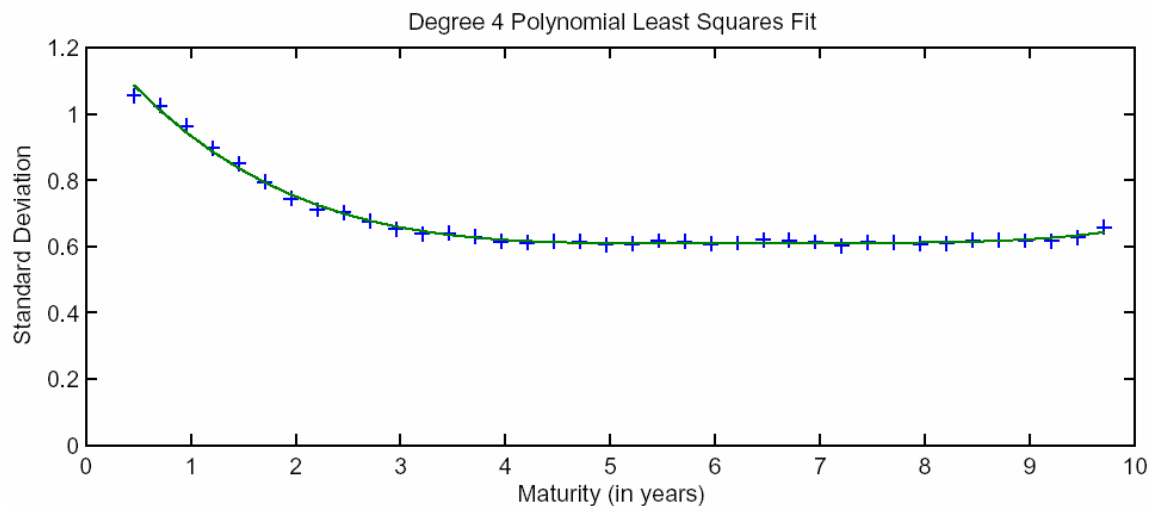
The correlation coefficient tells how related the movements of the spot and futures interest rates are. This can be either a positive or negative number. A positive number suggests that the rates have a positive correlation. For example, when one rate increases, the other also increases. The standard deviations for the futures and spot rates are the calculated standard deviations across different maturities.

## METHOD

This study was conducted to show that interest rate futures are volatile and do not experience parallel shifts across maturities. To accomplish that, the study used the Eurodollar futures contract, because it is the most actively traded futures in the market. In order to observe the volatility of Eurodollar futures contracts, historical data was collected. The quarterly data came from the *Wall Street Journal* and included data from 1997-2001. The maturities ranged from 5.5 months to 10 years. All of these futures contracts come due on the third Wednesday of each month in which futures contracts mature. All futures contracts mature in March, June, September, or December. The data was collected on December 31<sup>st</sup>, April 1<sup>st</sup>, July 1<sup>st</sup>, and October 1<sup>st</sup>. This offered 20 data points for each time to maturity observation. However, due to the fact that the 1<sup>st</sup> of the month was not always a trading day and that the third Wednesday of each month could vary by as much as seven days, the maturity of each data point may vary by several days. Given the fact that the data shows such a distinct pattern, the slight discrepancy in maturities seems immaterial. Standard deviations were then calculated for each time to maturity. Those numbers were then analyzed using regression analysis and were given a curve of best fit. This curve represents a polynomial function that serves as a model for the data. This function allows one to extrapolate and find the approximate volatility for a maturity that was not represented in the sample.

## RESULTS

Figure 1 illustrates the term structure of the futures contract volatility. Time to maturity is on the horizontal axis and the volatility, measured by standard deviation of the futures contracts, is on the vertical axis.



**Figure 1**

$$\sigma_f = 1.25 - 0.404*t + 0.0939*t^2 - 0.00963*t^3 + 0.000367*t^4 \quad (2)$$

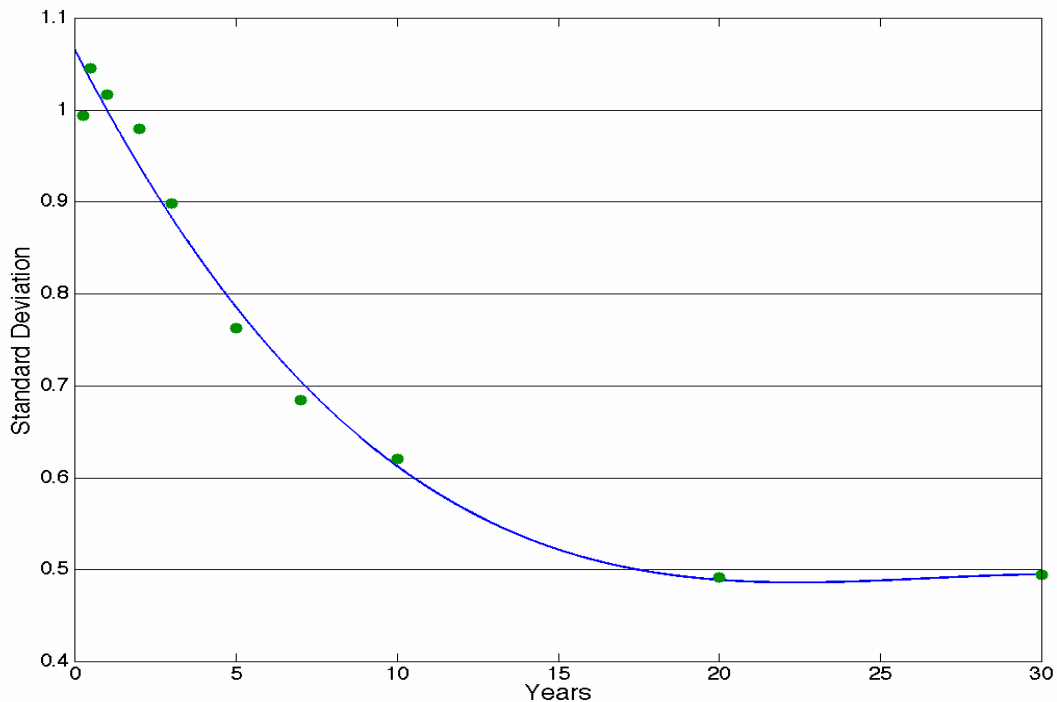
Figure 1, given by equation (2) illustrates that volatility decreases rapidly for the first two years. The decrease becomes more gradual until the term structure becomes flat for the duration at 3 years until maturity.

## IMPLICATIONS

To illustrate the implications for the hedge ratio, it helps to first look at a “textbook” example. In a typical classroom example of hedging with futures it is assumed that the standard deviations of the futures rate and spot rate remain constant across all maturities. With that assumption, once the hedge ratio is in balance, it will remain balanced until maturity of the position. In reality, it is much more complex.

Assume that the standard deviation of the spot rate remains constant, and as this study suggests, the futures rate is non-constant. As the figure suggests, the hedge ratio may remain stable for longer maturity futures, but may become quickly unbalanced for short maturity futures as the volatility changes quickly with time. Adjustments to the hedge will be required to maintain a balanced hedge ratio.

The reality is that both the standard deviations of the spot and futures interest rates are non-constant. This paper shows that there is volatility in the futures standard deviation and Stuart (2002) shows that there is also volatility in the standard deviation of the spot rate (see Figure 2). This complicates the maintenance of a balanced hedge ratio. However, there are a few ways of dealing with the added complexity.



**Figure 2**

The slope of the graph at any point represents the rate of change of the standard deviation. This research suggests that this rate of change is not constant, but dynamic. In order to keep a balanced hedge, one would find it useful to hedge with futures contracts so that the slope of the volatility of the hedging device is equal to the slope of the volatility of the spot rate. In theory, that would make the rate of change of the volatilities the same. That could be accomplished through the use of only one maturity of futures contracts or a combination of different maturities that, together equal the slope of the spot position. This should keep the hedge ratio balanced for a longer time period.

The following example will clarify this point. As stated, the slope of the futures and spot rate graphs represent the change in the standard deviation divided by the change in time. It is common convention to assume that the standard deviations of the spot and futures rates remain constant across all maturities. This research and that of Stuart (2002) suggests otherwise. The equation for the curve of the spot rate (Stuart 2002) is,

$$\sigma_s = 1.0653 - 0.068281*t + 0.0026235*t^2 - 0.000032512*t^3 \quad (3)$$

In order to find the slope at a given maturity, the derivative needs to be calculated. It is as follows,

$$\sigma_s' = -0.068281 + 0.005247*t - 0.000097536*t^2 \quad (4)$$

The formula for the curve representing the volatility of the futures position is,

$$\sigma_f = 1.25 - 0.404*t + 0.0939*t^2 - 0.00963*t^3 + 0.000367*t^4 \quad (2)$$

The rate of change for the futures position can be calculated with this equation,

$$\sigma_f' = -0.404 + 0.1878*t - 0.02889*t^2 + 0.001468*t^3 \quad (5)$$

This research, coupled with that of Stuart (2002) can suggest the futures maturity whose rate of change in volatility will match that of the underlying position ( $\sigma_s' = \sigma_f'$ ). Given the assumption of a spot contract with a one year maturity ( $t = 1$ ), the slope of this position is equal to  $-0.06313$  [ $\sigma_s'(1)$ ]. The maturity of the futures position that matches this slope is 2.96 years ( $\sigma_s'(t) = 0.06313$ ,  $t = 2.96$  years). By selling futures with maturity equal to 2.96 years, the volatility of the spot will change at the same rate as the volatility of the future. This should keep the hedge balanced longer than alternative hedge maturities. Assuming that the correlation between the spot rate and the futures rate is one, the hedge ratio will be equal to the following,

$$h = 1(.9996/.6553)$$

$$h = 1.525$$

In theory, by selling 1.525 futures contracts with maturities of 2.96 years, the hedge should remain balanced the longest.

## CONCLUSION

As Figure 1 suggests, futures volatility does vary over time, generally decreasing with maturity. With a little intuition, one could infer that there is more inherent risk involved as maturity approaches. It has been shown that the standard deviations of the interest rates of Eurodollar futures contracts are not constant. They change over time and do so in varying degrees. When this is coupled with the fact that there is also volatility in the spot rate (Stuart 2002), it becomes more difficult to maintain a balanced hedge. The combination of the findings can suggest how to maintain balance in the hedge.

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