PROPOSAL NARRATIVE

An optimal design of a new freeway system

1. Proposal Narrative

A. Abstract

This research proposal is to find an algorithm that will identify the most cost-effective and ecofriendly routes to build a set of freeways connecting the new power plant and the edge of the city. To do this, we will study double-weighted graphs, as well as methods for finding the cheapest and ecofriendly paths in a double-weighted graph. In a double-weighted graph, each vertex represents a neighborhood. If two vertices are connected by a double-weighted edge, then that means a strip of freeway can be built to connect the two neighborhoods, where the first weight is the cost of building the freeway and the second weight is the number of trees to be cut down. Our goal is to first find the minimum-cost set of freeways that connects all the vertices, then the most ecofriendly route that minimizes the number of trees needing to be cut down. The methods involved include Kruskal's algorithm and Prim's algorithm. Furthermore, by studying the two algorithms, we will create a new algorithm that can find the optimal route to build our freeway system, which will take both the cost and the damage to the environment into account.

B. Background/Statement of the Problem/Significance of the Project

A double-weighted graph contains vertices and double-weighted edges, where the weights on an edge can stand for the distance between the two adjacent vertices and the average time in hours to travel between the two adjacent vertices, where they are separated by a comma (see Figure 1 as an example).

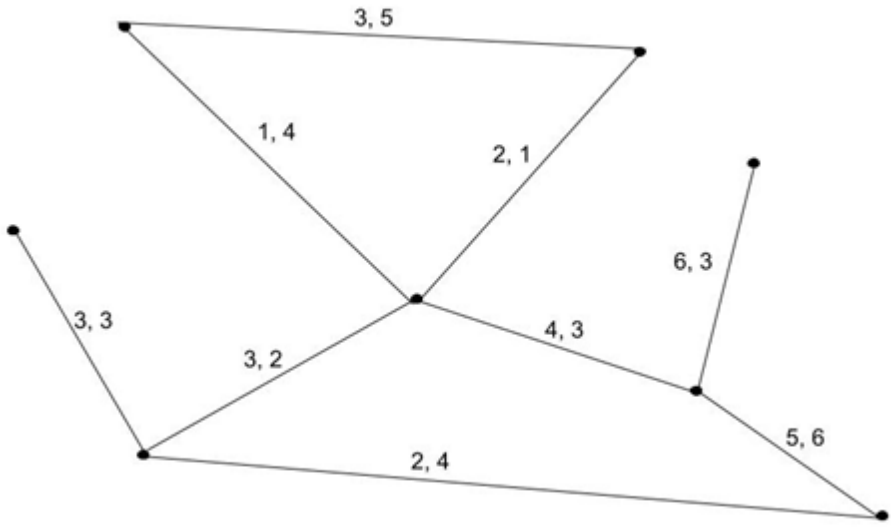

Figure 1

Double-weighted graphs are very useful in modeling the connection of pieces of road as seen on a map. This is because graphs can be made in which vertices are addresses and edges are roads. For each double-weighted edge, the first weight will be the cost to build that road and the second is the number of trees (in thousands) that would have to be cut down. This would have to be calculated for every edge of the graph, and would depend on the survey of the land, distance, and cost of materials.

Imagine a company like Foxconn moving into Milwaukee. They will need to develop the surrounding area to connect their manufacturing plant to the city in order for supplies and workers to come to the plant. Each strip of freeway to potentially be built has a specific cost to build and large number of trees to be cut down. If Foxconn were to build all potential strips of land, the cost would be much higher, and many more trees would be sacrificed. To avoid this, they should choose one route that leads from the plant to the city. This will allow for Foxconn to still be connected to the city while saving money and trees. It can be difficult if there are many different paths to build though; how can Foxconn be sure they picked the least expensive and harmful route? This is where the application to graph theory comes in.

Once Foxconn has mapped out all possible strips of freeway to build (a through l) that connect the plant (L) to the city (W), they can create a graph of the freeway strips and the cost to

build and the number of trees needing to be cut down for each. Assume the graph of the freeway system looks like this (see figure 2). Foxconn would be able to use the algorithm that we create to find which direct path is the most cost effective to build or the most Earth friendly or the best combining both of the two factors.

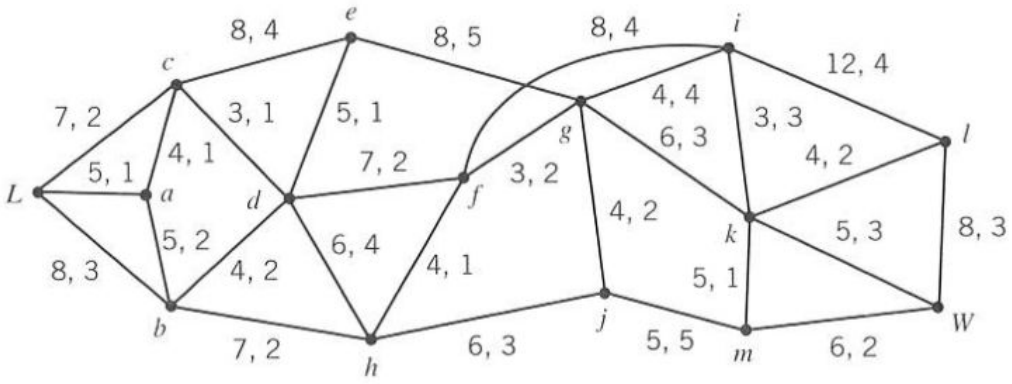

Figure 2

Our goal was to find first the minimum-cost set of freeways connecting all the vertices together by using Kruskal's algorithm as mentioned above. Secondly, we wanted to find the set of connecting freeways that minimizes the number of trees cut down using Prim's algorithm as mentioned above. These two algorithms were studied so that we have had a better understanding to create our algorithm. We were able to complete these two goals by coding up and running the algorithms in our computer program in Fall 2018.

For further work, we will combine the two algorithms above to create a new one that will consider both the cost and the number of trees to cut down. Then an ideal outcome will be to lower the cost as possibly as we can and meanwhile to reduce the number of trees to cut down. Using our new algorithm that we will create, Foxconn will find the ideal set of freeways that will minimize both the cost and the number of trees cut down.

References:

1. Introduction to Graph Theory, by G. Chartrand and P. Zhang
1. Algorithmic Graph Theory, by A. Gibbons
1. Graphs, Networks and Algorithms, 3rd edition, by D. Jungnickel

1. Applied Combinatorics, by A. Tucker

C. Objectives

The main outcome of this research is to code two algorithms that will find one path that is the most cost effective and the other path that is the most Earth friendly. Furthermore, our final goal is to find a plan that can guarantee it's both the most cost effective and the most eco-friendly, i.e., it's to minimize both the cost and the number of trees to cut down.

- The first objective of this research project was to fully learn and understand the existing algorithms associated with different weights in weighted graphs, such as Kruskal's algorithm and Prim's algorithm. This objective has been achieved in Fall 2018.
- The second objective of this research project was to create two algorithms using Java coding that will find which path has the lowest cost and which path has the lowest number of trees to cut down in the graph. Therefore, the optimization problem of Foxconn can be partially solved. The second objective has been achieved too.
- The third objective of this research project will be to create an optimal algorithm that can minimize both the cost and the number of trees to cut down.

D. Research Methods

This project relies on theoretical work, and so our research will utilize many concepts, theorems, and proofs from graph theory. I will be having regular meetings with Dr. Yan to stay in close contact.

The first objective was accomplished by studying existing algorithms that are relating to weights in graphs, such as the aforementioned Kruskal's algorithm and Prim's algorithm.

The second objective was accomplished by using our knowledge of graphs and weights in graphs, as well as various coding techniques from CS 120 and CS 220.

The third objective can be accomplished by adjusting the algorithms from the second objective, where the difficult part is how to quantify the damage to the environment in terms of the number of trees to cut down. We expect to create a function of two variables as our new weight function so that we will have a new single-weighted graph to deal with.

E. Final Products and Dissemination

I will present my research for the Math and Stats Club. Then, I will also present at the 23nd annual Celebration of Student Research and Creativity, as well as submit my research in the UWL Journal of Undergraduate Research.

F. Budget justification

We have estimated that this project will take about 100 hours. I will work on the project objectives throughout the spring and summer semesters.