Using Rolling Circles to Generate Caustic Envelopes Resulting from Reflected Light

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When light rays radiating outward from a source $S$ (called a radiant) in the plane are reflected off of a curved mirror (a smooth curve) in that plane, the reflected light rays produce a brightly lit curve called a caustic which is the envelope of those reflected rays. Pictured below on the left are two such caustics resulting from radiants inside an elliptical mirror. The radiants are at the dots in the first and third quadrants and the resulting caustics are concentrated mostly in the opposite quadrant. On the right we see parallel light from a radiant at infinity being reflected from a parabolic mirror producing the caustic that loops around the focus of the parabola.

We will discuss envelopes in general, but mostly concentrate on the caustic envelopes. A new theorem will be demonstrated: If the caustic envelope $E$ is a result of light from radiant $S$ reflected off of mirror curve $\alpha$, then there is another curve $\beta$ and a circle $C$ that rolls along $\beta$ without slipping such that there is a point on $C$ that traces out the caustic $E$ as the circle rolls.

Hence every caustic can be created in manner similar to the way the famous cycloid is traced out by a point on the rim of a wheel as the wheel rolls along a line. There will be many nice pictures and animations demonstrating the theorem in action. The talk uses only a little algebra, trigonometry and calculus. There will be some definitions and a little history, but its mainly going to be fun pictures and animations!

Friday, March 27th
Time: 3:30-4:30pm  
Centennial Hall 2205