Consider the additive group $\mathbb{Z}_8 = \{0, 1, 2, \ldots, 7\}$. Now assign each number in $\mathbb{Z}_8$ a color from the set \{red\}. Hold that thought.

A $k$-term arithmetic progression in $\mathbb{Z}_n$ is a set of distinct elements of the form

$$a \mod n, a + d \mod n, a + 2d \mod n, \ldots, a + (k - 1)d \mod n$$

where $d \geq 1$ and $k \geq 2$. An $r$-coloring of $\mathbb{Z}_n$ is a function $c : \mathbb{Z}_n \rightarrow \{1, 2, \ldots, r\}$. We say such a coloring is exact if $c$ is surjective (uses all of the colors at least once) and an arithmetic progression is rainbow if the image of the progression is injective (all of the numbers in the AP have different colors).

Now, in your coloring of $\mathbb{Z}_8$ above do you have any rainbow 3-AP’s? Of course not. What if you colored the $\mathbb{Z}_8$ using only the colors \{red, blue\}, are there any rainbow 3-AP’s? Of course not again. All right, this one might be harder. Can you color $\mathbb{Z}_8$ with \{red, blue, green\}, use each color at least once and avoid making a rainbow 3-AP?

The anti-van der Waerden number, $aw(\mathbb{Z}_n, k)$, denotes the smallest number of colors with which elements of the cyclic group of order $n$ can be colored and still guarantee there is a rainbow arithmetic progression of length $k$. In this setting, arithmetic progressions can “wrap around.” We will discuss results and open questions with $k = 3$.

Friday, September 12th
3:30pm, Room 1401

All Welcome to Attend
Centennial Hall